Homework Assignment 22

Assigned Tuesday March 9; due Friday March 19, 2PM, at SS 1071

Required reading. All of Spivak’s Chapter 22.

To be handed in. From Spivak Chapter 22: 1 (odd parts), 2 (odd parts), 5, 13.

Recommended for extra practice. From Spivak Chapter 22: 1 (even parts), 2 (even parts), 9, 27, 28, 29.

Just for fun. For some constant number $c$, consider the function $f_c(x) = 4cx(1 - x)$. Let $A$ be the set of all pairs $(c, y)$ so that $0 \leq c \leq 1$ and $y$ is a limit of a subsequence of the sequence $f_c(\frac{1}{2}), f_c(f_c(\frac{1}{2})), f_c(f_c(f_c(\frac{1}{2}))), \ldots$. Write a computer program to draw the set $A$ in the plane whose axes are $c$ and $y$, and if your program and picture are nice, they’ll find their place on this class’ web site.

It’s a hard one, but it’s well worth it. The set $A$ is way more complex than you would expect, with parts that scream “structure” and parts that scream “mess”. If you’ve ever heard the word “chaos” in a mathematical context before, this is it. And if you’ve ever seen pictures of the beautiful “Mandelbrot Set”, our $A$ is a close relation.