(1) Let \( V = \mathbb{R}^4 \) and let \( e_1, e_2, e_3, e_4 \) be its standard basis. Let \( \mathcal{A}^3(\mathbb{R}^4) \) be the space of alternating 3-tensors on \( \mathbb{R}^4 \). Let \( T \) be a 2 tensor on \( V \) given by \( T(u, v) = 2u_1v_2 + 3u_1v_1 - 5u_3v_4 \). Let \( S \) be a 1-tensor on \( V \) given by \( S(u) = 2u_1 + u_2 - 3u_4 \). Express \( \text{Alt}(T \otimes S) \) in the standard basis of \( \mathcal{A}^3(\mathbb{R}^4) \).

(2) Let \( T \) be a \( k \)-tensor on \( R^n \). Prove that \( T \) is \( C^\infty \) as a map \( \mathbb{R}^{nk} \to \mathbb{R} \).

(3) Let \( M \) be a union of \( x \) and \( y \) axis in \( \mathbb{R}^2 \). Prove that \( M \) is not a \( C^1 \) manifold.

(4) Prove that \( S_+^2 = \{(x, y, z) \in \mathbb{R}^3| \text{ such that } x^2 + y^2 + z^2 = 1, z \geq 0 \} \) is a manifold with boundary.

(5) Let \( c: [0, 1] \to (\mathbb{R}^n)^n \) be continuous. Suppose that \( c^1(t), \ldots, c^n(t) \) is a basis of \( \mathbb{R}^n \) for any \( t \).

Prove that \( (c^1(0), \ldots, c^n(0)) \) and \( (c^1(1), \ldots, c^n(1)) \) have the same orientation.

(6) Let \( C \) be the triangle in \( \mathbb{R}^2 \) with vertices \((0, 0), (1, 2), (-1, 3)\)

Compute \( \int_C x + y \).

(7) Let \( e_1, e_2 \) be a basis of a vector space \( V \) of dimension 2. Let \( T \in \mathcal{L}^2(V) \) be given by \( e_1^* \otimes e_1^* + e_2^* \otimes e_2^* \).

Prove that \( T \) can not be written as \( S \otimes U \) with \( S, U \in \mathcal{L}^1(V) \).

(8) Let \( U \subset \mathbb{R}^n \) be open. Let \( f, g: U \to \mathbb{R} \) be continuous and \( |f| \leq g \). Suppose \( \int_U^\text{ext} g \) exists.

Prove that \( \int_U^\text{ext} f \) also exists.