(1) Let $S$ be a rectifiable subset of the $xz$ plane in $\mathbb{R}^3$ such that $\text{Cl}(S) \subset \{x > 0\}$. Let $V$ be a solid obtained by rotating $S$ around $z$ axis. Prove that $V$ is rectifiable and $\text{vol}(V) = 2\pi \int_S x$.

*Hint:* Use cylindrical coordinates.

(2) Let $n > 1$. Give an example of an $n \times n$ matrix $A$ which preserves volume but is not orthogonal.

(3) Let $A$ be an $n \times n$ matrix with $\det A = 0$ and $S \subset \mathbb{R}^n$ be a rectifiable subset.

Prove that $A(S)$ has volume 0.

(4) Let $v_1, \ldots, v_n$ be $n$ vectors in $\mathbb{R}^n$. Let $B$ be an $n \times n$ matrix with $B_{ij} = \langle v_i, v_j \rangle$.

Prove that $\det B \geq 0$ and $\text{volP}(v_1, \ldots, v_n) = \sqrt{\det B}$.

(5) Let $f: \mathbb{R} \to \mathbb{R}$ be given by $f(x) = |x|$. Prove that the graph of $f$ is not a $C^1$ manifold.