Let \( A, B \) be \( n \times n \) real matrices. Define \( \langle A, B \rangle = \text{tr}(A \cdot B^t) \).

Prove that this defines an inner product on the space of all \( n \times n \) matrices.

(2) Let \( \{C_i\}_{i \in I} \) be a family of subsets in a set \( X \). Prove that
\[
X \setminus (\bigcup_i C_i) = \bigcap_i (X \setminus C_i)
\]

(3) Show that the norm \( || \cdot ||_{\infty} \) on \( \mathbb{R}^n \) satisfies the triangle inequality
\[
||x + y||_{\infty} \leq ||x||_{\infty} + ||y||_{\infty}
\]
for any \( x, y \in \mathbb{R}^n \).

(4) Show that the norms \( || \cdot || \) and \( || \cdot ||_{\infty} \) on \( \mathbb{R}^n \) satisfy
\[
||x||_{\infty} \leq ||x|| \leq \sqrt{n} \cdot ||x||_{\infty}
\]
for any \( x \in \mathbb{R}^n \).

(5) Prove that metrics coming from \( || \cdot || \) and \( || \cdot ||_{\infty} \) on \( \mathbb{R}^n \) define the same open sets.

Hint: Use Problem (4).

(6) Show that interior of any set is an open set.

(7) Prove that a set \( A \subset \mathbb{R}^n \) is closed if and only if it contains all its boundary points.

Extra Credit Problem (to be written up and submitted separately)

Suppose \( v_1, \ldots v_{k+1} \) are nonzero vectors in \( \mathbb{R}^n \) such that \( \angle v_i v_j > \pi/2 \) for any \( i \neq j \).

Show that \( v_1, \ldots v_k \) are linearly independent.