(1) Find a mistake in the following "proof".

**Claim:** \(1 + 2 + \ldots + n = \frac{1}{2}(n + \frac{1}{2})^2\) for any natural \(n\).

We proceed by induction on \(n\).

a) The claim is true for \(n = 1\).

b) Suppose we have already proved the claim for some \(n \geq 1\). We need to prove it for \(n + 1\).

We know that \(1 + 2 + \ldots + n = \frac{1}{2}(n + \frac{1}{2})^2\). Then \(1 + 2 + \ldots + n + (n + 1) = \frac{1}{2}(n + \frac{1}{2})^2 + (n + 1) = \frac{1}{2}(n^2 + n + \frac{1}{4} + 2(n + 1)) = \frac{1}{2}(n^2 + 3n + \frac{9}{4}) = \frac{1}{2}(n + \frac{3}{2})^2 = \frac{1}{2}((n + 1) + \frac{1}{2})^2.

This verifies the claim for \(n + 1\) and therefore the claim is true for all natural \(n\).

(2) Find \(6^{100}\mod 22\).

(3) Let \(a, b, c\) be natural numbers such that \(gcd(a, b) = 1\). Suppose \(a\) divides \(c\) and \(b\) divides \(c\).

Prove that \(ab\) also divides \(c\).

(4) Let \(p = 3, q = 5\) and \(E = 11\). Let \(N = 3 \cdot 5 = 15\). The receiver broadcasts the numbers \(N = 15, E = 11\). The sender sends a secret message \(M\) to the receiver using RSA encryption. What is sent is the number \(R = 3\).

Decode the original message \(M\).

(5) Mark True or False. If true explain why, if false give a counterexample.

(a) The product of any two irrational numbers is irrational.

(b) For any prime \(p\) we have \(((p - 1)!)^2 \equiv 1\mod p\).