(1) Find the formula for the sum

$$1 \cdot 2 - 2 \cdot 3 + 3 \cdot 4 - \ldots + (2n-1) \cdot (2n) - (2n) \cdot (2n+1)$$

and prove it by mathematical induction.

(2) Find the remainder when $6^{100}$ is divided by 28.

(3) Find the integer $a$, $0 \leq a < 37$ such that $(34!)a \equiv 1 \pmod{37}$.

(4) Let $n = pq$ where $p$, $q$ are distinct odd primes. Find the remainder when $\phi(n)!$ is divided by $n$.

(5) Find all integer solutions of the equation

$$34x + 50y = 22$$

(6) Let $(a, m) = 1$ Prove that for any $c$ there exists $b$ such that $ab \equiv c \pmod{m}$. 