(1) Prove by mathematical induction that $n^3 + 5n$ is divisible by 6 for any natural $n$.
(2) Find the remainder when $7^{101}$ is divided by 101.
(3) Find the integer $a$, $0 \leq a \leq 20$ such that $13a \equiv 1 \pmod{20}$.
(4) Prove that if $m \equiv 1 \pmod{\phi(n)}$ and $(a, n) = 1$ then $a^m \equiv a \pmod{n}$, where $\phi$ is Euler’s function.
(5) Suppose $3^{3^{100}}$ is written in ordinary way. What are the last two digits?
(6) Prove that $\sqrt[3]{\frac{2}{7}}$ is irrational.