Functions and Inverses – Problems

1. (a) If $f(x)$ is an invertible function and $f(2) = -5$, what is $f^{-1}(-5)$?
(b) If $f(x)$ is an invertible function and $f(0) = 2$, what is $f \left( f^{-1}(0) \right)$?
(c) Let $f(x) = x^3$. At how many points do the graphs of $y = f(x)$ and $y = f^{-1}(x)$ intersect?

2. Find the domain of the following functions:
   (a) $f(x) = \sqrt{2x + 7} - \frac{x}{3x - x^2}$
   (b) $f(x) = \frac{3}{3x - x^2} + \frac{\sqrt{9 - x^2}}{3}$

3. (a) Let $f(x) = \frac{7x + 1}{9 - 2x}$. Find $f^{-1}(x)$.
(b) Does the relation described by $xy = 7y + 8$ define a function in the variable $y$? What about a function in the variable $x$?
(c) Under which circumstances does the function $f(n) = n!$ (n factorial) have an inverse function? Where doesn’t the inverse exist? Explain.
(d) Let $h(x) = x^2 - 2x + 8$ and $g(x) = \sqrt{x}$. Write an expression for $(g \circ h \circ g)(x)$ in terms of $x$.
(e) Let $f(x) = x + 1$ and $g(x) = \frac{1}{x}$. Compute $(f \circ g)^{-1}(2)$.

4. (a) Which of the following relations are functions of $q$:
   
   $w = q + 1$ , $q = \frac{2q + 1}{w}$ , $wq = -27$.
   
   (b) Find the inverse function $f^{-1}$ for each of the following.
   
   $f(x) = 3x + 2$ , $x^2 + 6x + 3$ for $x \leq -3$ , $f(x) = \frac{x + 3}{5x - 1}$.
   
   (c) Let $f(x) = \frac{2x}{1-x}$. Find all real numbers $x$, if any, for which $f(-x) = 2f(x)$.
   
   (d) Let $f(x) = \sqrt{x} + 1$, $g(x) = x^2 - x$, and $h(x) = \frac{1}{x - 2}$. Evaluate and simplify the following
      
      $f \left( g(x) \right)$ , $(h \circ g)(x)$ , $f \left( g(h(x)) \right)$ , $(g \circ h \circ g)(x)$.
5. Give a different function for each of the following questions so that the function has exactly the given domain and range.

(a) Domain = \(\mathbb{R}\), Range = \(\mathbb{R}\).
(b) Domain = \((-\infty,0) \cup (0,\infty)\), Range = \((-\infty,0) \cup (0,\infty)\).
(c) Domain = \(\mathbb{R}\), Range = \{4\}.
(d) Domain = \((0,1) \cup (1,2) \cup (2,\infty)\), Range = \((3,\infty)\).

Exponentials and Logarithms

1. (a) If \(f(x) = 2^x\), then what is \(f^{-1}(1024)\)?

   (b) Find the inverse function \(f^{-1}(x)\) of \(f(x) = e^{4x-2}\).

2. \(\frac{e^{7x-1}}{e^{x-1}} = (e^6)^7\). Find \(x\).

3. For how many values does \(e^x = 0\)? What does that tell us about the value of \(\ln(0)\)?

4. Evaluate the following

   \[ \log_3 \left( \frac{1}{27} \right) , \quad \log_4 \left( \frac{1}{4} \right) , \quad \log_{25} (\sqrt{5}) , \quad \ln(1). \]

5. Are there any solutions to the equation

   \[ \ln(x^3 - 2x^2 - x + 2) - \ln(x + 1) - \ln(x - 2) = -\ln(2) ? \]

   Why or why not?

6. Evaluate the following.

   (a) \(\frac{e^{1+2\ln 7)^2}}{(7^{1+\ln 7})^2(73)^{1+\ln 7}}\)

   (b) \(\frac{\log_5 25 - \log_5 \frac{1}{10}}{3^{\log_3 2} - e^{\ln 8}}\)

   (c) \(\frac{\log_6 4 - 2 \log 25(5) + \log_6 9}{\log_5 (3^{1-3} - 3^{-2} - 27^{-1})}\)

7. Solve the following equation: \(e^{\ln x + \ln(x+4)} = 5\).

8. Do the graphs of \(y = e^x\) and \(y = \ln x\) intersect? If so, where? If not, how do you know?
9. Which of the following are equal to \( \frac{1}{2} \):

\[ e^{\ln(0.5)}, \quad e^{-\ln(2)}, \quad \ln(1) - \ln(2), \quad \frac{3e^0}{6} \]

Write the equation of the horizontal asymptote to the graph \( y = e^x \) and the equation of the vertical asymptote to the graph of \( y = \ln x \). Use the definition of “inverse functions” to explain how the equations of the two asymptotes are related.

10. Solve the following equation:

\[ 2^x = 5^{9x-2}. \]

11. If the population of rabbits on a particular island is given by the equation \( P = 10 \cdot 2^t \), where \( t \) is the time (in year), find the initial population of rabbits on the island.

Then find how many years it will take for the population to reach 1000.