1. Each of given 100 numbers was increased by 1. Then each number was increased by 1 once more. Given that the first time the sum of the squares of the numbers was not changed find how this sum was changed the second time.

**Solution.** Given that the sum of the squares did not change when we added 1 to each number, we have
\[(a_1 + 1)^2 + (a_2 + 1)^2 + \cdots + (a_{100} + 1)^2 - a_1^2 - a_2^2 + \cdots + a_{100}^2) = 0 \text{ or } (2a_1 + 1) + (2a_2 + 1) + \cdots + (2a_{100} + 1) = 0.\] Therefore, we have
\[a_1 + a_2 + \cdots + a_{100} = -50.\] If we increase each number by 1 once more, the sum of squares will be change by
\[(a_1 + 2)^2 + (a_2 + 2)^2 + \cdots + (a_{100} + 2)^2 - (a_1^2 + a_2^2 + \cdots + a_{100}^2) = 4(a_1 + 4) + 4(a_2 + 4) + \cdots + 4(a_{100} + 4) = 4 \times (-50) + 400 = 200.

2. Mother baked 15 pasties. She placed them on a round plate in a circular way: 7 with cabbage, 7 with meat and one with cherries in that exact order and put the plate into a microwave. All pasties look the same but Olga knows the order. However she doesn’t know how the plate has been rotated in the microwave. She wants to eat a pasty with cherries. Can Olga eat her favourite pasty for sure if she is not allowed to try more than three other pasties?

**Solution.** Denote the cherry pasty by 0, the cabbage pasties by 1, . . . , 7 and the meat pasties by −1, . . . , −7. If Olga does not get the cherry pasty on her first try, it must be either a cabbage pasty or a meat pasty. On her second try Olga takes the 4-th pasty from the first one in the direction of the cherry pasty. She gets either the cherry pasty 0, or the cabbage pasty 1,2,3, or the meat pasty −1, −2, −3.

On her last try Olga takes the second pasty from her second try in the direction of the cherry pasty and gets either the cherry pasty 0, or the cabbage pasty 1, or the meat pasty −1. Hence, after at most three tries Olga knows the position of the cherry pasty for sure.

3. The entries of a 7 × 5 table are filled with numbers so that in each 2 × 3 rectangle (vertical or horizontal) the sum of numbers is 0. For 100 dollars
Peter may choose any single entry and learn the number in it. What is the
least amount of dollars he should spend in order to learn the total sum of
numbers in the table for sure?

**SOLUTION.** Let $S$ be a total sum of the numbers in the ta-
ble. Let Peter divide the table into 6 rectangles as shown
on the picture (two rectangles overlap on a marked en-
try). Then $S = 0 \times 5 + (0 - x)$ where $x$ is the value in
the marked entry he would pays for. Since a $7 \times 5$ table
cannot be split into $2 \times 3$-rectangles without overlapping
(or holes) Peter cannot find $S$ for free.

4. Point $L$ is marked on side $BC$ of triangle $ABC$ so that $AL$ is twice as
long as the median $CM$. Given that angle $ALC$ is equal to $45^\circ$ prove that
$AL$ is perpendicular to $CM$.

**SOLUTION.**

Let $O$ be a point of intersection of $KL$ and $MC$. Let $K$ be a point of
intersection of $AL$ and a line drawn through $M$ parallel to $BC$. Then $AK =
KL$. Since $MK$ is parallel to $CL$, triangles $KMO$ and $OLC$ are similar
and we have $KO/(KL - KO) = MO/(MC - OM)$. Since $KL = MC$,
$KO = OM$ and each of triangles $OKM$ and $OCL$ is isosceles and therefore
$\angle OCL = \angle OLC = 45^\circ$. Hence, $\angle COL = 90^\circ$.

5. Ali Baba and the 40 thieves want to cross Bosporus strait. They made a
line so that any two people standing next to each other are friends. Ali Baba
is the first; he is also a friend with the thief next to his neighbour. There is a single boat that can carry 2 or 3 people and these people must be friends. Can Ali Baba and the 40 thieves always cross the strait if a single person cannot sail?

**SOLUTION.** (Vassily Kapustin, grade 8, Poplar Bank P.S.) If the number of thieves $n$ is even then $A, T_1, T_2$ sail to Europe, and $A, T_1$ sail back leaving $T_2$ in Europe. Then $T_3, T_4$ sail to Europe, $T_2, T_3$ sail back and now $T_4$ is in Europe and everybody else is in Asia. Continuing this process we end up with $T_n$ (the last thief in line) in Europe, and $A, T_1, \ldots, T_{n-1}$ with the boat in Asia.

If the number of thieves $n$ is odd then $A, T_1, T_2$ sail to Europe, and $A, T_2$ sail back leaving $T_1$ in Europe. Then $T_2, T_3$ sail to Europe, $T_1, T_2$ sail back and now $T_3$ is in Europe and everybody else is in Asia. Continuing this process we end up with $T_n$ in Europe, and $A, T_1, \ldots, T_{n-1}$ with the boat in Asia.

We can see that after applying described operation the last thief in the line will be in Europe and all the remained gang in Asia. We are again in conditions of the original problem but the number of thieves decreased by 1. Therefore, we apply this process several times until we get only $A, T_1, T_2$ in Asia. Then the trio sail to Europe and join the gang.