International Mathematics
TOURNAMENT OF THE TOWNS

Junior O-Level Paper               Fall 2009.¹

1. Is it possible to cut a square into nine squares and colour one of them
   white, three of them grey and five of them black, such that squares of
   the same colour have the same size and squares of different colours will
   have different sizes?

2. There are forty weights: 1, 2, . . . , 40 grams. Ten weights with even
   masses were put on the left pan of a balance. Ten weights with odd
   masses were put on the right pan of the balance. The left and the right
   pans are balanced. Prove that one pan contains two weights whose
   masses differ by exactly 20 grams.

3. A cardboard circular disk of radius 5 centimetres is placed on the table.
   While it is possible, Peter puts cardboard squares with side 5 centime-
   tres outside the disk so that:
   (1) one vertex of each square lies on the boundary of the disk;
   (2) the squares do not overlap;
   (3) each square has a common vertex with the preceding one.
   Find how many squares Peter can put on the table, and prove that the
   first and the last of them must also have a common vertex.

4. We only know that the password of a safe consists of 7 different digits.
   The safe will open if we enter 7 different digits, and one of them matches
   the corresponding digit of the password. Can we open this safe in less
   than 7 attempts?

5. A new website registered 2000 people. Each of them invited 1000 other
   registered people to be their friends. Two people are considered to
   be friends if and only if they have invited each other. What is the
   minimum number of pairs of friends on this website?

Note: The problems are worth 3, 4, 4, 5 and 5 points respectively.

¹Courtesy of Andy Liu