Juniors
(Grades up to 10)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper

Fall 2004.

1 [3] Is it possible to arrange integers from 1 to 2004 in some order so that the sum of any 10 consecutive numbers is divisible by 10?

2 [4] A box contains red, green, blue, and white balls; 111 balls in total. It is known that among any 100 of them there are always balls of all 4 colors in mention.
   Find the minimal number $N$ such that among any $N$ balls there are always balls of at least 3 different colors.

3 [4] A country consists of several cities; some of them are connected by Direct Express buses (each route connects two cities without intermediate stops).
   Mr. Poor bought one ticket for every bus route while Mr. Rich bought $n$ tickets for every bus route (a ticket allows a single one-way travel in either direction). Both Mr. Poor and Mr. Rich started from town $A$. Mr. Poor finished his travel in town $B$ using up all his tickets without buying extra ones. Mr. Rich, after using some of his tickets, got stuck in town $X$: he cannot leave it without buying a new ticket. Prove that $X$ is either $A$ or $B$.

4 [5] A circle and a straight line with no common points are given. With compass and straightedge construct a square with two adjacent vertices on the circle and two other vertices on the line (it is known that such a square exists).

5 [5] Find the number of ways to decompose 2004 into a sum of positive integers (one or more) that all are “approximately equal”.
   Decompositions obtained from one another by permutations are not considered as different.
   Two numbers are called *approximately equal* if their difference is at most 1.

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1Your total score is based on the three problems for which you earn the most points. Points for each problem are shown in brackets [ ].