Seniors
(Grades 11 and up)

International Mathematics
TOURNAMENT OF THE TOWNS

O-Level Paper

Spring 2003.

1 [3] 2003 dollars are placed into \( N \) purses, and the purses are placed into \( M \) pockets. It is known that \( N \) is greater than the number of dollars in any pocket. Is it (always) true that there is a purse with less than \( M \) dollars in it?

2 [3] 100-gon made of 100 sticks. Could it happen that it is not possible to construct a polygon from any lesser number of these sticks?

3 [4] Point \( M \) is chosen in \( \triangle ABC \) so that the radii of the circumcircles of \( \triangle AMC, \triangle BMC, \) and \( \triangle BMA \) are no smaller than the radius of the circumcircle of \( \triangle ABC \). Prove that all four radii are equal.

4 [5] In the sequence 00, 01, 02, 03, \ldots, 99 the terms are rearranged so that each term is obtained from the previous one by increasing or decreasing one of its digits by 1 (for example, 29 can be followed by 19, 39, or 28, but not by 30 or 20). What is the maximal number of terms that could remain on their places?

5 [5] Prove that one can cut \( a \times b \) rectangle, \( \frac{b}{2} < a < b \), into three pieces and rearrange them into a square (without overlaps and holes).

Keep the problem set.
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