1 Let \( j \) and \( m \) be numbers selected by \( J \) and \( M \) respectively. Note that \( j \mid 2002 \); otherwise \( J \) would know that \( m = 2002 - j \). Also \( j \neq 2002 \); otherwise \( m = 1 \) (since \( m \neq 0 \)). So, \( j \leq 1001 \). Further, the same is true for \( m \). In addition, \( M \) knows that \( j \leq 1001 \). Therefore, \( m = 1001 \) (otherwise \( M \) would know \( j = 2002 : m \)).

So, \( m = 1001 \) is the only possible solution. One can check that it works.

2 Let \( N \) be the number of students in the class, \( M \) the number of the problems, \( P \) the number of passed students, \( H \) the number of hard problems. According to definition “a problem is hard” if it has not been solved by at least \( rN \) students; where \( r = \frac{2}{3}, \frac{3}{4}, \frac{7}{10} \) in (a), (b), (c).

Also, according to definition “a student passes” if he solves at least \( rM \) problems.

- **a)** It is possible. Consider a class consisting of students \( S_1, S_2, S_3 \) and set of problems \( P_1, P_2, P_3 \). Let \( S_1 \) solve \( P_1 \) and \( P_3 \), \( S_2 \) solve \( P_2 \) and \( P_3 \) and \( S_3 \) solved neither \( P_1 \) nor \( P_2 \). Then \( S_1, S_2 \) pass and \( P_1, P_2 \) are hard problems.

- **b)** It is impossible. Let us write down the results of the test (“+” or “−”) into \( N \times M \) table.

Let passed students be on the top and hard problems on the left of the table. Let us estimate \( K_+ \) and \( K_- \), the numbers of “+” and “−” in the table. First,

\[
K_+ \geq \text{(number of “+” got by students who passed)} \geq P \times rM \geq r^2MN
\]

and

\[
K_- \geq \text{(number of “−” got for hard problems)} \geq H \times rN \geq r^2MN.
\]

Then \( MN = K_+ + K_- \geq 2r^2 MN \) which is impossible for \( r = \frac{3}{4} \).

- **c)** It is impossible. Arguments of (b) do not work here since \( 2r^2 \leq 1 \). Now we denote by \( K_+ \) and \( K_- \) the numbers of “+” and “−” in the top-left \( P \times H \) sub-table. Then

\[
K_+ \geq \text{(minimal number of “+” for hard problems got by students who passed)} \geq P \times \frac{4}{7}H
\]

(a student cannot pass if he solves less than \( \frac{4}{7}H \) of hard problems even if he solves all the easy problems, the number of which does not exceed \( \frac{3}{7}M \)). On the other hand,

\[
K_- \geq \text{(minimal number of “−” got by students who passed for hard problems)} \geq H \times \frac{4}{7}P.
\]

So, \( PH = K_+ + K_- \geq \frac{8}{7} PH \) which is impossible.
Let us assume that such point $B$ exists (separated from $A$ by each line). Then segment $AB$ intersects all the lines and therefore ray $[BA]$ originated at $B$ has no points of intersection beyond $A$. Therefore, $A$ belongs to unbounded region.

Now, assume that $A$ belongs to unbounded region. Our region is convex, bounded by two rays and maybe several segments. Note, that these rays are divergent. Therefore, one can draw a ray, originated at $A$ and lying inside of our region. Without any loss of the generality we can assume that this ray is not-parallel to any of the lines; otherwise we can rotate it slightly. Then the opposite ray (originated at $A$) intersects all the lines and any point $B$ beyond the last point of intersection satisfies the condition.

Since function $\cos x$ is a monotone decreasing on $(0, \pi/2)$ we have $(x - y)(\cos x - \cos y) \leq 0$ (equality holds only for $x = y$). Also $(x - z)(\cos z - \cos x) \leq 0$ and $(y - z)(\cos y - \cos z) \leq 0$. Adding these inequalities we get

$$2(x \cos x + y \cos y + z \cos z) \leq (y + z) \cos x + (x + z) \cos y + (y + x) \cos z$$

and therefore

$$3(x \cos x + y \cos y + z \cos z) \leq (x + y + z)(\cos z + \cos y + \cos x)$$

which implies our inequality.

Let $\{a_k\}$ be our sequence. Note that $1 \leq a_{k+1} - a_k \leq 9$. Then the segment $[9 \ldots 989, 9 \ldots 999]$ contains a term of our sequence; $a_k = 9 \ldots 99r$. If $r$ is even than $a_k$ is even. If $r$ is odd then $a_{k+1}$ must be odd.