INSTRUCTIONS

Non-programmable calculator permitted; no other aids allowed.

Present your solutions to all of the following questions in the exam booklets supplied. The marks for each question are indicated in parantheses.

TOTAL MARKS: 100.

1. (15 marks: 5 marks each) Find the following:
   (a) \( \int xe^x \, dx \)
   (b) the length of the curve with parametric equations
   \[ x = \cos t; \quad y = \sin t; \quad z = t^{3/2} \]
   for \( 0 \leq t \leq 1. \)
   (c) a unit tangent vector to the curve \( \mathbf{r} = \sin^{-1}t \mathbf{i} + \ln(t + 1) \mathbf{j} \) at the point for which \( t = 0. \)

2. (15 marks) Find the general solution to each of the following differential equations:
   (a) (5 marks) \( \frac{dy}{dx} = 3y + 5 \)
   (b) (10 marks) \( \frac{dy}{dx} + \frac{xy}{x^2 + 1} = \frac{1}{x} \)
3. (15 marks: 5 marks each) The following three parts are not related.

(a) Find the 5th degree Taylor polynomial of \( f(x) = \frac{x}{1-x} \) at \( x = 0 \)

(b) Find the interval of convergence of the power series \( f(x) = \sum_{n=0}^{\infty} \frac{n}{n^2 + 1}x^n \)

(c) Approximate the value of \( \int_{0}^{1/3} \sin x \frac{1}{x} \, dx \) correct to within 0.0001, and explain why your approximation is correct to within 0.0001

4. (15 marks) Consider the cardioid with polar equation \( r = 1 - \sin \theta \).

(a) (5 marks) Plot the cardioid, and label all \( x \) and \( y \) intercepts.

(b) (5 marks) Find the length of the cardioid.

(c) (5 marks) Find the area of the region within the cardioid.

5. (10 marks) Find the critical points of \( f(x, y) = 4x^3 - 6xy^2 + 3y^4 \) and at each critical point determine whether \( f \) has a relative maximum point, a relative minimum point, or a saddle point.

6. (10 marks) Do the following infinite series converge or diverge? Justify your answer.

(a) (3 marks) \( \sum_{n=1}^{\infty} \frac{n^2 + n - 1}{n^{5/2} - n^{3/2} + 4} \)

(b) (3 marks) \( \sum_{n=1}^{\infty} \frac{\sin(1/n)}{n} \)

(c) (4 marks) \( \sum_{n=1}^{\infty} \frac{2^n \ln n}{5^n \sqrt{n}} \)

7. (10 marks) Let \( f(x) = \frac{x^2}{(1+x^2)^2} \).

(a) (6 marks) Use the binomial series expansion for \( (1 + x^2)^{-2} \) to find the Maclaurin series for \( f(x) \) and its radius of convergence.

(b) (4 marks) What is the exact value of \( \sum_{n=1}^{\infty} \frac{n(-1)^n}{3^{2n}} \)?

8. (10 marks) Find \( \int_{1}^{\infty} \frac{x + 1}{x^2 + x^4} \, dx \).