You may only quote theorems we have covered in the class so far.

**Question 1.** Let $\mu \in \Omega^1(M, \mathbb{R})$ be a 1-form on a smooth manifold $M$ such that $d\mu = 0$ and $\mu(p) \neq 0$ for some $p \in M$. Show there exists a neighbourhood $U$ of $p$ and coordinates $(x_1, \ldots, x_n)$ in this neighbourhood such that $\mu = dx_1$ in $U$.

**Question 2.** Consider the differential 2-form on $\mathbb{R}^3$ given by

$$\rho = x dy \wedge dz + y dz \wedge dx + z dx \wedge dy,$$

and let $i : S^2 \hookrightarrow \mathbb{R}^3$ be the usual inclusion of the unit sphere. Show that $i^* \rho$ determines an orientation on $S^2$, and compute $\int_{S^2} i^* \rho$ with respect to this orientation.

**Question 3.** Computations of de Rham cohomology: justify all computations.

i) Determine the de Rham cohomology groups of $D^2 \setminus \{(0,0)\}$, where $D^2$ is the open unit disc in $\mathbb{R}^2$. Give differential forms representing a basis for each cohomology group. Which of these forms can be chosen to have compact support\(^1\)?

ii) Using Mayer-Vietoris, determine the de Rham cohomology groups of $D^2 \setminus \{p_1, \ldots, p_k\}$, for $\{p_i\}$ distinct points in $D^2$.

iii) Determine the de Rham cohomology groups of $\mathbb{R}^3 \setminus Z$, where $Z$ is the union of three distinct rays emanating from the origin.

iv) Using Mayer-Vietoris, determine the de Rham cohomology groups of $\Sigma_g$, the compact orientable surface of genus $g$.

**Question 4.** Computations of fundamental groups: give justification for all computations.

i) $S^{n-1}$ is naturally included in $S^n$ via the inclusion of $\mathbb{R}^n$ in $\mathbb{R}^{n+1}$; by composition this defines a natural inclusion $S^1 \subset S^n$. Compute $\pi_1(S^n \setminus S^1)$. Hint: write $S^3 = \mathbb{R}^3 \sqcup \{\infty\}$.

ii) By composing the above inclusions we also have $S^m \subset S^n$ for $m < n$. Compute $\pi_1(S^n \setminus S^m)$.

iii) The above inclusions induce inclusions $\mathbb{R}P^m \subset \mathbb{R}P^n$ for $m < n$; compute $\pi_1(\mathbb{R}P^n \setminus \mathbb{R}P^m)$.

**Question 5.** Give an example of a space with fundamental group $\mathbb{Z}_3 \times \mathbb{Z}_4$ (hint: build a cell complex)

**Question 6.** Give an example of a covering which is not normal (abnormal?)

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\(^1\) $\rho$ has compact support when the closure of $\{x : \rho(x) \neq 0\}$ is compact.