These homework problems are meant to expand your understanding of what goes on during class. Any you turn in will be graded and returned to you. Answers may or may not be posted on the web, depending on demand.

1. Can a finite figure have more than one rotational symmetry? (That is, can a finite figure have rotational symmetries with different centers of rotation?)

2. Recall that a regular polygon is a polygon whose sides are all the same length, and whose angles are all the same. Show that a regular $n$-sided polygon has precisely $n$ rotational symmetries and precisely $n$ reflection symmetries.

3. Is there a finite figure whose symmetry group is something other than $C_N$ or $D_N$? If not, then we have classified the symmetry groups of finite figures. That is, we have listed all possible symmetry groups.

4. We saw in class that, on the square, both

\[ r^5mr^{-3}mr^2 = r^2 \]
\[ r^2mr^{-3}mr^5 = r^2. \]

That is, we have written both sides of the (true) first equation backwards and obtained the (also true) second equation. The question is: is this always the case? That is, if you write both sides of a true equation backwards, do you obtain a true equation? You may answer this for the square or for an $n$-sided regular polygon.

5. (Hamoon’s conjecture) As a follow-up to the previous question: if we re-arrange the exponents of the various $r$ terms on each side of a (true) equation to obtain a new equation, must this new equation also hold? You may answer this for the square or for an $n$-sided regular polygon.

6. Prove that the identity of a group is unique.

7. Let $a$ and $b$ be elements of a group $G$. Prove that if $a^{-1} = b^{-1}$, then $a = b$.

8. Please make a model cube, tetrahedron, icosahedron, or dodecahedron. (You may make more than one, but please make at least one.) There are links on the class web page to paper models, or you may build frame models with, say, sticks or toothpicks.