1. Suppose that $A, B$ are $n \times n$ matrices. Assume that there exists an invertible $n \times n$ matrix $Q$ such that $Q^{-1}AQ = B$. Prove that there exists a vector space $V$ and a linear operator $T : V \to V$ such that $A$ and $B$ are both matrices for $T$ (with respect to two different bases).

2. Consider the matrix $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

   (a) Consider $A$ as a linear operator on $\mathbb{R}^2$ and find a basis for $\mathbb{R}^2$ consisting of eigenvectors for this linear operators.

   (b) Find an invertible matrix $Q$ such that $Q^{-1}AQ$ is diagonal.

3. Prove that the following conditions on a square matrix $A$ are equivalent.

   (a) $A$ is a scalar multiple of the identity matrix.

   (b) Every vector is an eigenvector for $A$.

   (c) $A$ is diagonalizable and has only one eigenvalue.

   (d) There are no matrices (other than $A$) which are similar to $A$.

4. For each of the following complex matrices $A$, determine if there exists a complex matrix $B$ such that $B^2 = A$. (Hint: use Jordan form.)
5. Recall that an $n \times n$ complex matrix $A$ is called nilpotent if 0 is its only eigenvalue. How many $5 \times 5$ nilpotent Jordan form matrices are there?