Problem 1.
Compute \((\sqrt{2}/2 + i\sqrt{2}/2)^{239}\).

Problem 2.
Compute \(\exp(\ln 2 + \pi i)\).

Problem 3.
Find all the solutions of the equation \(z^5 = 32\).

Problem 4.
Find \(\lim_{z \to 0} \frac{e^z - e^{-z}}{z}\).

Problem 5.
Show that the function \(u(x, y) = \sin x(e^y + e^{-y})\) is harmonic, and find its harmonic conjugate.

Problem 6.
Show that the function \(f(x + iy) = x^4 + y^4 + 1 - 2x^2 + 2y^2 - 6x^2y^2 + i(4x^3y - 4xy^3 - 4xy)\) is entire, and find its derivative.

Problem 7.
For a function \(f(z) = \frac{z + i}{z - i}\) find the level curve \(|f(z)| = 1\).

Problem 8.
Write the polynomial \(z^4 - z + 1\) at the Taylor form centered at \(i\).

Problem 9.
Find all the poles of the rational function \(f(z) = \frac{z + i}{z^3 - 2}\).

Problem 10.
Find \(\lim_{n \to \infty} \frac{\sinh(n)}{n}\).
Answers

Problem 1.
\[ \sqrt{2}/2 - i\sqrt{2}/2. \]

Problem 2.
\[ -2. \]

Problem 3.
\[ 2, 2e^{2\pi i/5}, 2e^{4\pi i/5}, 2e^{6\pi i/5}, 2e^{8\pi i/5}. \]

Problem 4.
\[ 2 - \text{use L'Hospital rule}. \]

Problem 5.
\[ \cos x(e^y - e^{-y}). \]

Problem 6.
\[ f' = 4x^3 - 4x - 12xy^2 + i(12x^2y - 4y^3 - 4y). \]

Problem 7.
\[ Imz = 0. \]

Problem 8.
\[ 2 - i - (4i + 1)(z - i) - 6(z - i)^2 + 4i(z - i)^3 + (z - i)^4. \]

Problem 9.
\[ 0, 1, e^{2\pi i/3}, e^{4\pi i/3}. \]

Problem 10.
\[ 0. \]