Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student. 
Instructions: Fill in the information on this page, and make sure your test booklet contains 
10 pages. In addition, you should have a multiple-choice answer sheet, on which you 
should fill in your name, number, tutorial time, tutorial room, and tutor’s name. 
This test consists of 10 multiple choice questions, and 4 written-answer questions. 
For the multiple choice questions you can do your rough work in the test booklet, but you 
must record your answer by circling the appropriate letter on the answer sheet with your 
pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or 
two answers for the same question is worth 0. For the written-answer questions, present 
your solutions in the space provided. The value of each written-answer question is indicated 
beside it.

ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME:

GIVEN NAME:

STUDENT NO:

SIGNATURE:

TUTORIAL TIME and ROOM:

REGCODE and TIMECODE:

T.A.’S NAME:

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PART A. Multiple Choice

1. [4 marks]

A nominal rate of 4% compounded quarterly is equivalent to what nominal rate compounded semi-annually?

A. 4.01%

B. 4.02%

C. 4.03%

D. 4.04% [4.02% (B)]

E. 4.05%

Let \( r = \text{nominal rate compounded semi-annually} \)

\[
(1 + \frac{r}{2})^2 = (1.01)^4
\]

\[
1 + \frac{r}{2} = (1.01)^2 = 1.0201
\]

\[
\frac{r}{2} = 0.0201
\]

\[
r = 0.0402
\]

2. [4 marks]

After 4 years sitting in an account with interest compounded semi-annually, an initial investment of $2000 grows into $2844.20. The nominal annual rate is closest to

A. 4.5%

B. 4.6%

C. 8%

D. 9%

E. 9.2% [0.089999 = r (D)]
3. [4 marks]

A person borrows $5000 today at 6% compounded semi-annually. If he pays $1000 one year from now and $2000 two years from now, then how much must he pay 3 years from now in order to completely pay off the loan?

A. $2000

B. $2572.60

C. $2722.95

D. $2121.80

E. $2120

\[
5000 = 1000(1.03)^{-2} + 2000(1.03)^{-4} + x(1.03)^{-6}
\]

\[
5000(1.03)^{6} = 1000(1.03)^{4} + 2000(1.03)^{2} + x
\]

In any case \( x = \$222.95 \)  

(C)

4. [4 marks]

Every day for one year (of 365 days), a student deposits one dollar into an account earning 11% per year compounded daily. At the beginning the account holds $0. How much is in the account after the last deposit?

A. $377.98

\[
S = 1 \times \sum_{i=0}^{365} \left(1 + \frac{11}{365}\right)^i
\]

B. $385.77

\[
= \left(1 + \frac{11}{365}\right)^{365} - 1
\]

C. $387.74

D. $405.15

E. $956.30

\[
= \$385.77 \quad \text{(B)}
\]
5. [4 marks]

Mr. Khaghani wants to make his retirement easy by setting up a perpetuity that gives him $1500 per month for groceries and weekly outings to a local jazz club. How much money must he put in to the fund if his money earns a nominal interest rate of 16% per year compounded monthly?

A. $140,215
B. $131,200
C. $122,750
D. $115,000
E. $112,500

\[
\begin{align*}
\gamma &= \frac{16}{12} \\
R &= rA \\
1500 &= \frac{16}{12} A \\
A &= \frac{12 \times 1500}{16} = \$12,500
\end{align*}
\]

6. [4 marks]

Maria takes out a $500,000 mortgage amortized over 30 years at an interest rate of 6% per year compounded semi-annually with monthly payments at the end of each month. The monthly payments are closest to:

A. $2974
B. $2998
C. $2959
D. $1396
E. $1986

\[
\begin{align*}
(1.03)^2 &= (1 + \frac{1}{12})^{12} \\
(1.03)^{-60} &= (1 + \frac{1}{12})^{-360} \\
500,000 &= RA_{360|12} \\
R &= \frac{500,000}{A_{360|12}} \\
&= \frac{500,000 \left( \frac{(1.03)^{\frac{1}{2}} - 1}{1 - (1.03)^{-60}} \right)}{1 - (1.03)^{-60}} \\
R &= \$2974.11
\end{align*}
\]
7. [4 marks]

Let \( A = \begin{bmatrix} 3 & 2 \\ 0 & -5 \end{bmatrix} \) and \( B = \begin{bmatrix} -1 & 0 \\ 7 & 11 \end{bmatrix} \).

Let \( A + 2B = C \) where the entries of \( C \) are \( C_{ij} \). Then \( C_{21} = \)

A. 1

\[ C_{21} = A_{21} + 2B_{21} = 0 + 2 \cdot 7 = 14 \]  \( \square \)

B. 2

C. 14

or \( A + 2B = \begin{pmatrix} 3 & 2 \\ 0 & -5 \end{pmatrix} + \begin{pmatrix} -2 & 0 \\ 14 & 22 \end{pmatrix} \)

\[ = \begin{pmatrix} 1 & 2 \\ 14 & 17 \end{pmatrix} = C \]

D. 16

E. 17

\( C_{21} = 14 \) as before

8. [4 marks]

Let \( A \) be a \( 7 \times 3 \) matrix, \( B \) be a \( 5 \times 7 \) matrix, and \( C \) be a \( 5 \times 3 \) matrix. Then only one of the following is defined. Which one?

A. \( A + B - C \) \( A, B \) and \( C \) are not the same shape: not defined

B. \( CB - A \) \( CB \) not defined

C. \( CA + B \) \( CA \) not defined

D. \( AB + C \) \( AB \) not defined

E. \( BA - C \) result is \( 5 \times 3 \) and \( C \) is indeed \( 5 \times 3 \)

so defined \( \square \)
9. \(4 \text{ marks}\)

If \(A = \begin{bmatrix} 2 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -2 & -1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & 2 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\),

\(C = IAB^T\), and the entries of \(C\) are \(C_{ij}\), then \(C_{22} =\)

A. \(-2\)

\[ C = AB^T \text{ since mult by } I \text{ doesn't change anything}. \]

B. \(-1\)

\[ \begin{pmatrix} 2 & 0 \\ -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \]

C. \(1\)

D. \(2\)

To get \(C_{22}\), take the 2nd row of \(AB^T\) and the 2nd column of \(AB^T\).

E. \(0\)

\[-1 \cdot 0 + 0 \cdot (-1) + 1 \cdot 2 = 2 = C_{22} \quad \text{(D)} \]

10. \(4 \text{ marks}\)

The system

\[
\begin{align*}
x - y + z &= 4 \\
2x + 3z &= 3 \\
2y + z &= 5
\end{align*}
\]

has

A. a unique solution with \(z = 5\)

B. a unique solution with \(z = 10\)

C. a unique solution with \(y = 1\)

D. infinitely many solutions

E. no solution

\[
\begin{pmatrix}
1 & -1 & 1 & 4 \\
2 & 0 & 3 & 3 \\
0 & 2 & 1 & 5
\end{pmatrix} \xrightarrow{\text{R}_1 \rightarrow -2\text{R}_1 + \text{R}_2} \begin{pmatrix}
1 & -1 & 1 & 4 \\
0 & 2 & 1 & -5 \\
0 & 2 & 1 & 5
\end{pmatrix} \xrightarrow{\text{R}_3 \rightarrow -\text{R}_2 + \text{R}_3} \begin{pmatrix}
1 & -1 & 1 & 4 \\
0 & 2 & 1 & -5 \\
0 & 0 & 0 & 10
\end{pmatrix} \text{ (E)}
\]
PART B. Written-Answer Questions

1. [15 marks]

John takes out a loan from a wealthy friend for $250,000 and promises to pay it back over 20 years. His friend insists that he pay back the loan in monthly installments, beginning at the end of the first month, and is charging 5% per year compounded monthly.

[7] (a) What is the amount John must pay his friend each month?

\[ 250,000 = R \left( \frac{1}{2^{10} \cdot 0.05} \right) \]

\[ R = \frac{250,000 \times 0.05}{1 - (1 + 0.05)^{-240}} \]

\[ R = \$1649.39 \]

[8] (b) After 10 years pass, John and his friend decide to renegotiate the loan. They agree to the following new terms in paying off the balance: the interest rate will be 5.5% per year compounded semi-annually and the payments will be every 6 months, with the first new payment at the end of the first 6 months. How much must John pay every 6 months?

Principal outstanding = \[ R \left( \frac{1}{2^{10} \cdot 0.05} \right) \]

\[ = 1649.39 \left[ 1 - \left(1 + \frac{0.055}{2}\right)^{-120} \right] \]

\[ = 8155,553,86 \]

If we call the new payments \( T \)

\[ 155,553.86 = T \left( \frac{1}{2^{10} \cdot 0.055} \right) \]

\[ T = 155,553.86 \times \frac{0.055}{2} \]

\[ T = \$10,215.49 \]
2. [15 marks]

Lina buys a bond with 9 semi-annual coupons remaining at an annual coupon rate of 4.6% at a price of $98.41 per $100 of face value. Find the annual yield to maturity of the bond.

(Your answer will be sufficiently accurate if the price comes out within $0.50 of the actual price.)

\[ 98.41 = 100(1.01)^{-9} + 2.30 \times P \]

Since the price is less than $100, the semi-annual yield must be more than 0.023.

Let's try 0.03:

\[ 100(1.03)^{-9} + 2.30 \times P = 94.55 \text{ much too low} \]

\$100 was closer.

So, 0.03 is much too high.

Try 0.026  \[ P = \$ 99.62 \] (only $0.79 off but not close enough)

Price is too low, so yield is still too high. Try 0.024  \[ P = \$ 99.20 \] ($0.79 off)

Try 0.025  \[ P = \$ 98.406 \] close enough.

The annual yield to maturity is **5%**.

Of course you may have hit on 0.025 much faster. That would be OK too.
3. [15 marks]

Sophie has $4.40 in nickels (5¢), dimes (10¢), and quarters (25¢). She has four times as many dimes as quarters. She has a total of 40 coins. How many of each coin does she have?

(Hint: Set up a linear system of equations and solve by using matrix reduction.)

Let \( N = \# \) nickels
\( D = \# \) dimes
\( Q = \# \) quarters

\[
\begin{align*}
5N + 10D + 25Q &= 440 \\
D - 4Q &= 0 \\
N + D + Q &= 40
\end{align*}
\]

\[
\begin{bmatrix}
1 & 1 & 1 & |40 \\
0 & 1 & -4 & |0 \\
5 & 10 & 25 & |440
\end{bmatrix}
\]

\[
R_3 \rightarrow -5R_1 + R_3 \\
R_2 \rightarrow -5R_1 + R_2 \\
R_3 \rightarrow \frac{1}{2}R_3
\]

\[
\begin{bmatrix}
1 & 1 & 1 & |40 \\
0 & 1 & -4 & |0 \\
0 & 1 & 0 & |240
\end{bmatrix}
\]

Back substitution:
\[
Q = 6 \\
D = 40 - 24 = 16 \\
N = 40 - 40 - 6 = 10
\]

Or, complete reduction:

\[
\begin{bmatrix}
1 & 0 & 0 & |10 \\
0 & 1 & 0 & |24 \\
0 & 0 & 1 & |6
\end{bmatrix}
\]

\[
N = 10 \\
D = 24 \\
Q = 6
\]
4. [15 marks]
Consider the system of linear equations

\[
\begin{align*}
    x_1 + 2x_2 + x_3 &= 0 \\
    x_1 - x_2 + kx_3 &= 0 \\
    x_1 + x_2 - x_3 &= 0
\end{align*}
\]

where \( k \) is some constant.

[9] (a) For what values(s) of \( k \) is there a solution other than the obvious \( x_1 = x_2 = x_3 = 0 \)?

\[
\left( \begin{array}{ccc|c}
    1 & 2 & 1 & 0 \\
    1 & -1 & k & 0 \\
    1 & 1 & -1 & 0 \\
\end{array} \right) \xrightarrow{R_1 \rightarrow -R_3 + R_2} \left( \begin{array}{ccc|c}
    1 & 2 & 1 & 0 \\
    1 & -1 & k & 0 \\
    0 & 2 & k-1 & 0 \\
\end{array} \right) \xrightarrow{R_2 \rightarrow -R_3 + kR_3} \left( \begin{array}{ccc|c}
    1 & 2 & 1 & 0 \\
    0 & 2 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
\end{array} \right)
\]

If \( k+5 \neq 0 \), there is a unique solution \( x_1 = x_2 = x_3 = 0 \). To get other solutions, \( k+5 = 0 \).

\[ k = -5 \]

[6] (b) For the value(s) of \( k \) you found in (a), what is the most general solution to the system?

In case \( k = -5 \)

\[
\left( \begin{array}{ccc|c}
    1 & 2 & 1 & 0 \\
    0 & 2 & 0 & 0 \\
    0 & 0 & 0 & 0 \\
\end{array} \right)
\]

or complete reduction

\[
\left( \begin{array}{ccc|c}
    1 & 0 & -3 & 0 \\
    0 & 1 & 2 & 0 \\
    0 & 0 & 0 & 0 \\
\end{array} \right)
\]

Back substitution

\[
\begin{align*}
    x_1 &= -2x_3 \\
    x_1 &= -x_3 - 2x_2 = -x_3 + 4x_3 = 3x_3 \\
\end{align*}
\]

So

\[
\begin{align*}
    x_1 &= 3x_3 \\
    x_2 &= -2x_2 \\
\end{align*}
\]

as before.