Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 10 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor’s name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it.

ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME: ____________________________________________

GIVEN NAME: ____________________________________________

STUDENT NO: ____________________________________________

SIGNATURE: _____________________________________________

TUTORIAL TIME and ROOM: _________________________________

RECODE and TIMECODE: _________________________________

T.A.’S NAME: ____________________________________________

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PART A. Multiple Choice

1. [4 marks]
\[
\lim_{x \to \infty} \frac{5 + 4x - 3x^3 + 7x^3 - 2x^4 + 3x^5}{2x^5 + 7x^4 - 3x^3 + 2x^2 + x + 1} = \frac{\lim_{x \to \infty} \frac{5x^5 + 4x^4 - 3x^5 + 7x^4 - 2x^6 + 3x^6}{2x^5 + 7x^4 - 3x^5 + 2x^4 + x^2 + 1}}{\lim_{x \to \infty} (2x^5 + 7x^4 - 3x^5 + 2x^4 + x^2 + 1)}
\]
\[
= \frac{3}{2}
\]
A. undefined
B. \(\frac{3}{2}\)
C. 5
D. 3
E. \(\frac{3}{2}\)

2. [4 marks]
\[
f(x) = \begin{cases} 
  a + 3x^{-1} & \text{if } x < 1 \\
  2 & \text{if } x = 1 \\
  b x^{-1} & \text{if } x > 1 
\end{cases}
\]
and if \(f\) is continuous at all values of \(x\), then:
A. \(a = 2\) and \(b = 2\)
B. \(a = 1\) and \(b = 4\)
C. \(a = -1\) and \(b = 2\)
D. \(a = 2\) and \(b = 4\)
E. \(a = -1\) and \(b = 4\)

\[
\lim_{x \to 1^-} f(x) = a + 3x^{-1} \quad \lim_{x \to 1^+} b x^{-1}
\]
\[
= a + 0 = a
\]
\[
\lim_{x \to 1^-} f(x) = b x^{-1}
\]
\[
\lim_{x \to 1^+} f(x) = b x^{-1}
\]
\[\text{So, } a = 2 = b x^{-1} \quad \text{and} \quad b = 4\]
3. [4 marks]
If \( y = (5x^3 - 1) \ln x \), then at \( x = 1 \)
\[
\frac{dy}{dx} = 15x^2 \ln x + \frac{5x^3 - 1}{x}
\]
A. 0
B. 15
C. 4
D. 6
E. 9

4. [4 marks]
If \( f(x) = \frac{x^2 + x}{\sqrt{x + 1}} \), \( f''(3) = \)
A. \( \frac{17}{4} \)
B. \( \frac{17}{4} \)
C. \( \frac{13}{4} \)
D. \( \frac{11}{4} \)
E. \( \frac{11}{4} \)
5. [4 marks]

Let \( f(x) = \sqrt{1 + 2g(x)^2} \), \( g(1) = 2 \), and \( g'(1) = 6 \). Then \( f'(1) = \)

A. \( 3 \) \[ p'(x) = \frac{1}{2 \sqrt{1 + 2g(x)^2}} \cdot 2 \cdot 2g(x)g'(x) \]

B. 18

C. \( 8 \)

D. \( \frac{4}{3} \)

E. \( \frac{2}{3} \)

\[ p'(1) = \frac{2 \cdot 2 \cdot 6}{\sqrt{1 + 2 \cdot 2^2}} = \frac{24}{3} = 8 \] \( \square \)

6. [4 marks]

Let \( f(x) = 2^{x^4} - 1 \). The slope of the tangent line to the graph of \( y = f(x) \) at \((1,4)\) is

A. 48

B. \( 21^2 \)

C. \( 48 \ln 2 \)

D. \( 4 \ln 2 \)

E. 6

\[ \frac{1}{f} f'(x) = 12x^3 \ln 2 \]

\[ f'(1) = 12 \ln 2 \]

\( f'(1) = 48 \ln 2 \) \( \square \)

Alternatively:

\[ f'(x) = \frac{3x^9}{2} \ln 2 \cdot 12x^3 \]

\[ f'(1) = 2 \ln 2 \cdot 12 = 48 \ln 2 \] as before
7. [4 marks]

Let \( f(x) = (x^2 + 5)^{\sqrt{x}} \) and \( x > 0 \). Then \( f'(x) = \)

A. \( \left( \frac{\ln(x^2 + 5)}{2\sqrt{x}} + \frac{\sqrt{x}(2x)}{x^2 + 5} \right) (x^2 + 5)^{\sqrt{x}} \)

B. \( \sqrt{x}(x^2 + 5)^{\sqrt{x} - 1} \)

C. \( (x^2 + 5)^{\sqrt{x}}(2x) \left( \frac{1}{2\sqrt{x}} \right) \)

D. \( \left( \frac{1}{2\sqrt{x}} \right) \left( \frac{2x}{x^2 + 5} \right) \)

E. \( \frac{\ln(x^2 + 5)}{2\sqrt{x}} + \frac{\sqrt{x}(2x)}{x^2 + 5} \)

8. [4 marks]

If \( C(I) = \sqrt{I} \) is the total spent on consumption depending on national income \( I \), then when \( I = 25 \), marginal propensity to save is

A. 0.9

B. 0.1

C. 0.8

D. 0.2

E. 0.5

\[ S = I - C \]

\[ \frac{dS}{dI} = 1 - \frac{dC}{dI} \]

\[ = 1 - \frac{1}{2\sqrt{I}} \]

\[ = 1 - \frac{1}{2 \sqrt{25}} \]

\[ = 1 - \frac{1}{10} = 0.9 \quad \text{(A)} \]
9. (4 marks)
\[
\lim_{x \to 1} \frac{x^{3/2} - 1}{x + 1} = \lim_{x \to 1} \frac{\frac{3}{2}x^{1/2} - \frac{1}{2}}{2} = \frac{1}{6}
\]

A. undefined
B. \(\frac{1}{2}\)
C. \(\frac{1}{3}\)
D. \(\frac{1}{6}\)
E. 1

10. (4 marks)
If \(x_1 = 1\) is used as a first estimate for a root of \(f(x) = x^5 + x - 1 = 0\), then Newton's method yields the third estimate (to 5 decimal places) \(x_3 = \)

A. 0.77262

\[x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\]

B. 0.76438

C. 0.78597

\[x_{n+1} = x_n - \frac{x_n^5 + x_n - 1}{5x_n^4 + 1}\]

D. 0.83333

E. 0.80591

\[x_3 = \frac{5}{6}\]

\[x_3 = \frac{4 \cdot (\frac{5}{6})^5 + 1}{5 \cdot (\frac{5}{6})^4 + 1} \approx 0.76438 2115\]

B
PART B. Written-Answer Questions

1. [10 marks]

Solve the following inequality for \( x \).

\[
\frac{(x - 3) \ln x}{e^x - 5} \leq 0
\]

The function is only defined for \( x > 0 \) (because of \( \ln x \)) and for \( e^x - 5 \neq 0 \), i.e., \( x \neq \ln 5 \approx 1.61 \).

The function is 0 only at \( x = 3 \) and \( x = 1 \).

So, on the following intervals, the function is continuous and \( \neq 0 \):

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<td>(+)</td>
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<td>((\ln 5, 3))</td>
<td>(-)</td>
</tr>
<tr>
<td>((3, \infty))</td>
<td>(+)</td>
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</table>

Since at \( x = \frac{1}{2} \):

\[
\frac{\frac{5}{2} \ln \left( \frac{1}{2} \right)}{e^\frac{1}{2} - 5} = \frac{\frac{5}{2} \ln 2}{\frac{1}{\sqrt{e}} - 5} = -
\]

since \( \ln 2 > 0 \) and \( \frac{1}{\sqrt{e}} < 5 \)

at \( x = \frac{3}{2} \):

\[
\frac{-\frac{3}{2} \ln \left( \frac{3}{2} \right)}{e^\frac{3}{2} - 5} = -
\]

since \( \ln \frac{3}{2} > 0 \) and \( e^{\frac{3}{2}} > 20 \)

at \( x = 2 \):

\[
\frac{-\ln 2}{e^2 - 5} < 0
\]

and at \( x = \) very large, everything is \( > 0 \).

Answer: \([0, 1] \cup (\ln 5, 3]\)

Alternatively \( 0 < x \leq 1 \) or \( \ln 5 < x \leq 3 \)
2. \{18 \text{ marks}\}
Evaluate the following limits or show they don't exist. (Show your work.)

(a) \[ \lim_{x \to 1^-} \frac{|x - 1|}{x^2 - 3x + 2} \]

\[ x \to 1^- \Rightarrow x \leq 1 \Rightarrow |x - 1| = 1 - x \]

We have \[ \lim_{x \to 1^-} \frac{1-x}{x^2 - 3x + 2} = \lim_{x \to 1^-} \frac{1-x}{(x-1)(x-2)} = \lim_{x \to 1^-} \frac{1}{x-2} = 1 \]

or: \[ \lim_{x \to 1^-} \frac{-1}{2x-3} = 1 \]

(b) \[ \lim_{x \to \infty} (4x + e^x)^\frac{2}{x} \]

Let \[ y = (4x + e^x)^\frac{2}{x} \]

\[ \ln y = 2 \frac{\ln(4x + e^x)}{x} \]

\[ \lim_{x \to \infty} \frac{\ln(4x + e^x)}{x} = \lim_{x \to \infty} \frac{2e^x}{4 + e^x} = \lim_{x \to \infty} 2 = 2 \]

or: \[ \lim_{x \to \infty} \frac{2}{4e^x + 1} = \frac{2}{0+1} = 2 \]

\[ \ln y \to 2, \quad \text{so} \quad y \to e^2 \]

(c) \[ \lim_{x \to 2} \left[ \frac{1}{\ln(x-1)} - \frac{1}{x-2} \right] = \lim_{x \to 2} \left( \frac{x-2}{(x-2)\ln(x-1)} \right) \]

\[ = \lim_{x \to 2} \frac{1 - \frac{1}{x-1}}{\ln(x-1) + \frac{x-2}{x-1}} = \lim_{x \to 2} \frac{1}{\ln(x-1) + \frac{x-2}{x-1} + 1} \]

\[ \text{L'Hopital:} \quad \lim_{x \to 2} \frac{1}{\ln(x-1) + \frac{x-2}{x-1} + 1} = \frac{1}{2} \]

There are many other combinations of algebra + L'Hopital that got were correctly.
3. [17 marks]
Let \( p = \text{unit price}, q = \text{quantity}, \) and \( m = \text{number of employees}. \) The demand function is
\[
p = 10 + \frac{100}{q} \quad \text{and} \quad q = \frac{60m}{\sqrt{11 + m^2}}
\]
When \( m = 5, \) \( q = 300/6 = 50 \) and \( p = 12. \)

[4] (a) find the marginal revenue.
\[
r = pq = 10q + 100
\]
\[
\frac{dr}{dq} = 10
\]

[6] (b) find the point elasticity of demand. Is demand elastic, inelastic, or of unit
elasticity?
\[
\mu = \frac{dq}{dp} = \frac{p}{q} \frac{d}{dp} \left( \frac{100}{q^2} \right) = -\frac{p}{100} = -\frac{1200}{100} = -12
\]
\[
|\mu| = 12 > 1 \quad \text{so demand is elastic}
\]

[7] (c) find the marginal revenue product.
\[
\frac{dr}{dm} = \frac{dr}{dq} \frac{dq}{dm}
\]
\[
\frac{dq}{dm} = 60 \left[ \frac{11 + m^2 - 2m \cdot m}{(11 + m^2)^2} \right] = \frac{60}{36} \left[ 6 - \frac{25}{6} \right] = \frac{60 \cdot 11}{6 \cdot 36} = \frac{55}{18} \approx 3.0555
\]
\[
\frac{dr}{dm} \approx 30.555 \quad \text{or} \quad 30 \frac{5}{6}
\]
4. [15 marks]

[7] (a) If \(3x + 4y - x^2y^3 = 4\) defines \(y\) implicitly as a function of \(x\), find \(y'\) as a function of \(x\) and \(y\). Then find the equation to the tangent line defined by this curve at \(x = 3, y = 1\).

\[
3 + 4y' - 2xy^3 - 3x^2y^2y' = 0 \\
y' = \frac{2xy^3 - 3x^2y^2}{4 - 3x^2y^2} \quad \frac{6 - 3}{4 - 27} = \frac{-3}{23} \text{ at } (3,1).
\]

\[y - 1 = \frac{-3}{23}(x - 3)\]

is the equation of the tangent (i.e. at \((3,1)\)).

[8] (b) \(y \ln x + xe^y = 1\) defines \(y\) implicitly as a function of \(x\) near the point \(x = 1, y = 0\). Find \(y''\) at \(x = 1, y = 0\).

\[
y' \ln x + \frac{y}{x} + e^y + xe^y y' = 0 \quad \text{At } (1,0) \quad e^0 + y' = 0 \quad y' = -1
\]

\[
\left( y'' \ln x + \frac{y'}{x} \right) + \frac{y'}{x} + e^y + (e^y y' + xe^y y') = 0 \\
\text{At } x = 1, y = 0, y' = -1
\]

\[
y'' \cdot 0 + \left( \frac{-1}{1} \right) + \frac{(-1)}{1} - 5 + (-1) + (-1) + 1 - (-1)^2 + y'' = 0 \\
-4 + 1 + y'' = 0 \\
y'' = 3
\]