Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 10 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor's name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it.

ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME: ____________________________

GIVEN NAME: ____________________________

STUDENT NO: ____________________________

SIGNATURE: ____________________________

TUTORIAL TIME and ROOM: ____________________________

RECODE and TIMECODE: ____________________________

T.A.'S NAME: ____________________________

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FOR MARKER ONLY

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PART A. Multiple Choice

1. [4 marks]

If a $100 account with interest compounded annually grows to $119.03 in 5 years, what is its nominal annual interest rate (to the nearest 0.01%)?

A. 3.40%
B. 3.55%
C. 3.45%
D. 3.50%
E. 3.60%

Let \( r = \text{nominal annual interest rate} \).

\[
119.03 = 100(1+r)^5
\]

\[
r = \left( \frac{119.03}{100} \right)^{\frac{1}{5}} - 1
\]

\[
= 0.035436 \ldots 
\]

\[
= 3.55\% \quad (\text{B})
\]

2. [4 marks]

A 30-year loan with interest at 6% per year compounded monthly has monthly payments of $2098.43. The value of the loan is closest to:

A. $353,000
B. $325,000
C. $375,000
D. $350,000
E. $300,000

Let \( i = \text{monthly interest rate} \).

\[
i = \frac{0.06}{12} = 0.005
\]

\[
A = 2098.43 \times 360,000
\]

\[
= 2098.43 \left[ \frac{1 - (1.005)^{-360}}{0.005} \right]
\]

\[
= 350,000.53
\quad (\text{D})
A

3. [4 marks]

A fund is set up in order to donate $10,000 to a certain charity at the beginning of each year, in perpetuity. If the first donation is to be 1 year after the fund is set up and interest is 7% compounded annually, how much money (to the nearest dollar) should be used to set the fund up?

A. $140,400
B. $144,293
C. $137,062
D. $138,925
E. $142,857

\[
\begin{align*}
\text{In a perpetuity, if } A & \text{ is the amount and } R \text{ is the periodic payment and } r \text{ is the rate per period} \\
R &= rA \\
a & = \frac{R}{r} = \frac{10,000}{0.07} \\
&= 142,857.14 \\
\end{align*}
\]

or

\[
A = \lim_{n \to \infty} R \frac{1}{r} \left( \frac{1}{1 + r} \right)^n
= \lim_{n \to \infty} R \left( \left( \frac{1}{1 + r} \right)^n \right) = \frac{R}{r} \text{ as } n \to \infty
\]

4. [4 marks]

A person opens an account at the beginning of January, 2014 with a deposit of $100 and continues making deposits of $100 each at the beginning of each month of 2014, with the last deposit at the beginning of December, 2014. If interest is 6% compounded monthly, then at the beginning of January, 2015, the account will have

A. $1233.56
B. $1237.41
C. $1239.72
D. $1228.75
E. $1225.24

- This is an annuity due. There are many ways to get the value on Jan 1, 2015.
- One way:

  On Dec 1, 2014 the account will have

  100 $1.005
  So one month later

  \[
  1,005 \times 100 \times 1.005 \approx 1239.72 
  \]
5. [4 marks]

A family has a $400,000 mortgage amortized over 15 years with monthly payments (at the end of each month) at an interest rate of 3% per year compounded semi-annually. The first payment is at the end of the first month. The interest in the first payment is closest to

\[
(1+i)^{12} = (1.015)^2
\]

A. $993.81
B. $1000.00
C. $1762.33
D. $1768.53
E. zero because there is no interest in the first payment

\[
400,000 \times (1.015)^{12} - 1 = 400,000 \times 1.999997617 \approx \$993.81 \text{ (A)}
\]

6. [4 marks]

A $1000 loan is to be repaid by 2 equal payments: the first payment 1 year from now and the second payment 2 years from now. If interest is 5% compounded continuously, what is the amount of each payment?

A. $545.22
B. $532.35
C. $542.09
D. $548.40
E. $538.77

\[
1000 = xe^{-0.05} + xe^{-2(0.05)}
\]

\[
x = \frac{1000}{e^{-0.05} + e^{-1}}
\]

\[
\approx \$538.77 \text{ (E)}
\]
7. [4 marks]

Let \( A = \begin{bmatrix} 7 & 0 & 4 \\ 2 & 3 & -1 \\ 1 & 5 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 4 & -1 \end{bmatrix} \) and \( I \) be the \( 3 \times 3 \) identity matrix. If \( C = A^T B - 3I \), then \( c_{13} \) is

A. -2
B. 8
C. 0
D. 3
E. 23

\[
A^T B - 3I = \begin{pmatrix} 7 & 2 & 1 \\ 0 & 3 & 5 \\ 4 & -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 4 & -1 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}
\]

\[
= \begin{pmatrix} 1 & 8 & 8 \\ 5 & 26 & -2 \\ 0 & -2 & 3 \end{pmatrix} - \begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}
\]

\[
= \begin{pmatrix} -2 & 8 & 8 \\ 5 & 23 & -2 \\ 0 & -2 & 0 \end{pmatrix}
\]

\( c_{13} = 8 \) \( \boxed{B} \)

8. [4 marks]

If \( A \) is a \( 1 \times 3 \) matrix, \( B \) is a \( 2 \times 3 \) matrix and \( C \) is a \( 3 \times 1 \) matrix then the dimensions of the matrix given by

\[
2B(A^T + C) + 3B^T A C
\]

are

A. \( 1 \times 3 \)
B. \( 2 \times 3 \)
C. \( 3 \times 1 \)
D. \( 2 \times 1 \)
E. \( 3 \times 2 \)

\( 2 \times 1 + 2 \times 1 = 2 \times 1 \) \( \boxed{D} \)
9. [4 marks]

Which of the following augmented coefficient matrices represents a system of equations which has no solution?

A. \[
\begin{bmatrix}
1 & 2 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

B. \[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\]

C. \[
\begin{bmatrix}
1 & 5 & 3 \\
0 & 0 & 4
\end{bmatrix}
\]

D. \[
\begin{bmatrix}
1 & 9 & 0 & 0 & 0 & 5 \\
0 & 0 & 1 & 0 & 0 & 6 \\
0 & 0 & 0 & 1 & 1 & -3
\end{bmatrix}
\]

E. \[
\begin{bmatrix}
1 & 7 & 8 & 3 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

All matrices are in row-echelon form, and only C has a row of zeros with a non-zero right-hand-side. (C)

10. [4 marks]

The augmented coefficient matrix of a system of equations reduces to

\[
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 3 \\
0 & 1 & 0 & 0 & 1 & 2 \\
0 & 0 & 1 & 0 & 0 & -1
\end{bmatrix}
\]

The system has:

A. one unique solution
B. no possible solutions
C. a 1-parameter family of solutions
D. a 2-parameter family of solutions
E. a 3-parameter family of solutions

There are 5 variables and 3 equations remain; therefore, there are 2 parameters. (D)
PART B. Written-Answer Questions

1. [13 marks]

Teresa has $700 in a bank account earning an effective annual rate of 3%. In 10 years time, she will take all the money in the account and purchase a $1000 Government Bond that has been on the market for some time.

(a) How much is the bond selling for if she uses all of the money in the account to buy it?

\[ 700 \left( 1.03 \right)^{10} = \$940.74 \]

(b) At the time that she purchases the bond the following is true:
   i) The annual yield rate is 3%
   ii) The bond has semi-annual interest payments and the next one is due in 6 months
   iii) The bond matures in 6 more years

What is the annual coupon rate of the bond?

\[ t = 0.015 \quad r = \text{semi-annual coupon rate} \]
\[ n = 12 \]
\[ V = 1000 \]
\[ 940.74 = 1000 \left( 1.015 \right)^{-12} + r \times 1000 \left( 1 + \frac{1}{12}, 0.015 \right) \]

Solve for \( r \).

\[ r = 0.009567044 \]

\[ 2r \approx 0.01913 \]

Annual coupon rate \( 1.91\% \)
A $100,000 loan is amortized over 10 years at 4% per year with interest compounded annually and annual payments at the end of each year. If an extra payment of $10,000 (on top of the regular payment) is made at the end of the fourth year, how many payments are there altogether until the loan is paid off?

\[
\begin{align*}
100,000 &= R \left( \frac{1 - (1.04)^{-10}}{1.04} \right) \\
R &= \frac{100,000}{\left( \frac{1 - (1.04)^{-10}}{1.04} \right)} = \$12,329.09
\end{align*}
\]

Though we don't need to know this.

\[
\begin{align*}
100,000 &= 10,000(1.04)^{-4} + R \left( \frac{1 - (1.04)^{-10}}{1.04} \right) \\
100,000 &= 10,000(1.04)^{-4} + 100,000 \left( \frac{1 - (1.04)^{-10}}{1.04} \right) \\
100,000 &= 10,000(1.04)^{-4} + 100,000 \left( \frac{1 - (1.04)^{-10}}{1.04} \right)
\end{align*}
\]

Save for \( n \): e.g. divide by 100,000

\[
\begin{align*}
1 &= (1.04)^{-4} + \left( \frac{1 - (1.04)^{-10}}{1.04} \right) \\
\left( 1 - \frac{1}{1.04} \right) [1 - (1.04)^{-10}] &= [1 - (1.04)^{-n}] \\
(1.04)^{-n} &= \left\{ 1 - \left[ \frac{1 - (1.04)^{-10}}{1.04} \right] \right\} \\
n &= - \ln \left\{ \frac{(1.04)^{-4} + (1.04)^{-10} - (1.04)^{-10}}{1.04} \right\} \\
n \approx 8.97413 \quad \text{So 8 full payments} \\
&+ 1 \text{ small payment} \\
\end{align*}
\]
3. \(17\) marks

(a) Find all the solutions to the following system of equations using row reduction. [No marks for any other method.]

\[
\begin{align*}
2x_2 + 6x_3 & = 1 \\
x_1 + 2x_3 + 3x_4 & = -1 \\
3x_1 + 4x_3 + x_4 & = 5
\end{align*}
\]

\[
\begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4
\end{pmatrix}
= \begin{pmatrix} 
1 \\
-1 \\
5 \\
-4
\end{pmatrix}
\]

Row reduction:

\[
\begin{pmatrix}
0 & 2 & 6 & 0 \\
1 & 0 & 2 & 3 \\
3 & 0 & 4 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 2 & 3 \\
0 & 2 & 6 & 0 \\
0 & 0 & 14 & -4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 2 & 3 \\
0 & 1 & 3 & 0 \\
0 & 0 & 1 & -4
\end{pmatrix}
\]

Back substitution:

\[
x_3 = \frac{1}{4} (4 - 4x_4) \quad x_2 = \frac{1}{2} - 3x_3 = \frac{1}{2} - 3(\frac{1}{4} - x_4) = \frac{1}{2} + 3x_4
\]

\[
x_1 = -1 - 3x_4 - 2x_3 = -1 - 3x_4 - 2(\frac{1}{4} - x_4) = -\frac{1}{4} + x_4
\]

\[
\left\{\begin{array}{l}
x_1 = \frac{7}{2} + 5x_4 \\
x_2 = \frac{3}{2} + 12x_4 \\
x_3 = -4 - 4x_4 \\
x_4
\end{array}\right. 
\]

A 1-parameter family of solutions.

Or by complete reduction:

\[
\begin{pmatrix}
1 & 0 & 0 & -5 \\
0 & 1 & 0 & -12 \\
0 & 0 & 1 & 4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 0 & 7 \\
0 & 1 & 0 & 12
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 & 0 & 1 \\
0 & 1 & 0 & -4
\end{pmatrix}
\]

[?](b) Is there a solution to the system of equations from part (a) in which \(x_1 = 2\) and \(x_2 = 1\)? Using your solution in part (a), either find this solution or explain why there is none.

If \(x_2 = 1\), then the parameter \(x_4 = -1\).

But then \(x_2 = \frac{1}{2}\) not 1; so this is impossible.

There is no such solution.
4. [17 marks]

5/ (a) For which values of \( k \) does the matrix \[
\begin{bmatrix}
1 & 2 \\
-2 & k
\end{bmatrix}
\]
have an inverse, and what is \( A^{-1} \)? [Your answer will involve \( k \).]

\[
\begin{align*}
\begin{pmatrix}
1 & 2 \\
-2 & k
\end{pmatrix}
& \rightarrow
\begin{pmatrix}
1 & 2 \\
0 & k+4
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & -4 \\
0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & -4 \\
0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 0 \\
0 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 2 \\
0 & 1
\end{pmatrix}

\end{align*}
\]

\[k \neq -4, \text{there is no } A^{-1}\]

\[k \neq -4, \quad A^{-1} = \frac{1}{k+4} \begin{pmatrix} k-2 \\ 2 \end{pmatrix}\]

\((b)\) Let \( A = \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} \). Use your solution to part (a) to solve \( Ax = B \) where \( B = \begin{bmatrix} 4 \\ -1 \end{bmatrix} \). [No marks if you don't use your solution in (a)].

Note: with \( k = -3 \) in (a) we have our \( A \).

So \( A^{-1} = \frac{1}{-3+4} \begin{pmatrix} -3-2 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ 2 \end{pmatrix} \).

\( Ax = B \) so \( x = A^{-1} B \)

\[\begin{pmatrix}
1 & 2 \\
-2 & -3
\end{pmatrix}
\begin{pmatrix}
-x_A -2x_B \\
x_A -3x_B
\end{pmatrix}
= \begin{pmatrix}
4 \\
-1
\end{pmatrix}
= \begin{pmatrix}
-10 \\
4
\end{pmatrix}
\]

8/ (c) Two stores sell the same quantity of product A. The two stores also sell the same quantity of product B. The first store sells product A at $1 above cost (per unit) and product B at $2 per unit above cost. The second store sells product A at $2 below cost (per unit) and product B at $3 below cost. If the first store earns a total of $100 and the second store loses $150, how many units of each product did they sell? (Hint: use your solution to part (a) or (b)).

Let \( x_A = \text{no. of units of product A sold by each store} \) and \( x_B = \text{no. of units of product B sold by each store} \).

Store 1 earns \( x_A + 2x_B = 100 \)

Store 2 earns \(-2x_A - 3x_B = -150 \)

Notice: \( A (x_A) = (100) \) where \( A \) is the matrix in (b)

So \( (x_A, x_B) = A^{-1} (100) = (-3, -2) \)

\( (x_A, x_B) = (100) \quad \text{from (b)} \)

\( (x_A, x_B) = (-300 + 300) = (0) \quad \text{from (b)} \)

\( (x_A, x_B) = (200 - 150) = (50) \quad \text{from (b)} \)