Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 10 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor's name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it. ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME:  

GIVEN NAME:  

STUDENT NO:  

SIGNATURE:  

TUTORIAL TIME and ROOM:  

REPCODE and TIMECODE:  

T.A.'S NAME:  

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PART A. Multiple Choice

1. [4 marks]
   If interest is 8% compounded daily, how many days are required for a given principal to earn 4% in interest?
   \[
   P \left(1 + \frac{0.08}{365}\right)^n = P(1.04)
   \]
   
   A. 175  
   B. 179  
   C. 183  
   D. 177  
   E. 181  
   
   \[
   n \ln \left(1 + \frac{0.08}{365}\right) = \ln(1.04)
   \]
   
   \[
   n = \frac{\ln 1.04}{\ln \left(1 + \frac{0.08}{365}\right)} = 178.96 \ldots
   \]
   
   \[
   n \approx 179 \quad (B)
   \]

2. [4 marks]
   If interest is 10% compounded continuously, the effective annual rate is closest to
   \[
   1 + r_e = e^{10}
   \]
   
   A. 10.3%  
   B. 9.8%  
   C. 10.0%  
   D. 10.8%  
   E. 10.5%  
   
   \[
   r_e = e^{10} - 1 = 10.517 \ldots
   \]
   
   \[
   r_e \approx 10.5\% \quad (E)
   \]
3. [4 marks]

Monthly deposits are made into an account which earns 6% compounded monthly. The first 12 deposits are $500 each, but beginning with the 13th, each deposit is just $300. Just after the 24th deposit, the account (to the nearest dollar) has

\[ c = \frac{0.06}{12} \approx 0.005 \]

A. $10,372
B. $10,449
C. $10,249
D. $10,081
E. $10,168

There are many ways to do this:

\[ 500 S_{\frac{24}{1.005}} - 200 S_{\frac{12}{1.005}} \]

or

\[ 300 S_{\frac{24}{1.005}} + 200 S_{\frac{12}{1.005}} (1.005)^{12} \]

or

\[ 500 S_{\frac{12}{1.005}} (1.005)^{12} + 300 S_{\frac{12}{1.005}} \]

All come to

$10,248.8 \ldots$

so $\boxed{10,249}$

4. [4 marks]

If a $400,000 mortgage has monthly payments for 15 years and interest at 4.5% compounded semi-annually, then each payment is closest to

A. $3,030
B. $3,050
C. $3,070
D. $3,060
E. $3,040

Let \( i = \) monthly rate \( n = 12 \times 15 = 180 \)

\[ (1+i)^{12} = (1.0225)^2 \]

\[ 400,000 = Ra \frac{1}{180i} \]

\[ R = \frac{400,000i}{1 - (1+i)^{-180}} \]

\[ 400,000 \left[ (1.0225)^{12} - 1 \right] \]

\[ 1 - (1.0225)^{-30} \]

\[ \approx 3051.47 \]

closest to $\boxed{3050}$
5. [4 marks]
A bond selling for $187.91 has 16 semi-annual coupon payments remaining and an annual coupon rate of 6%. If the yield to maturity is 7%, then the face value of the bond is closest to:

A. $200
B. $195
C. $205
D. $175
E. $194

\[ 187.91 = \sqrt[16]{(1.035)^{-16}} + 0.03 \sqrt[16]{a_{0.035,16}} \]

\[ V = \frac{187.91}{(1.035)^{-16} + 0.03 a_{0.035,16}} \]

\[ V \approx 200, 004 \]

\[ V = 200 \quad (A) \]

6. [4 marks]
A bond with a face value of $100 matures in 5 years and has 10 semi-annual coupons of $5 each remaining. If the bond is selling for $145, then its annual yield to maturity is closest to

A. 1%
B. 2%
C. 3%
D. 4%
E. 6%

\[ P = 100 (1 + t)^{-10} + 5 a_{10 t} \] where
\[ t \text{ is semi-annual yield.} \]

\[ P = 145 \text{ is much larger than 100} \]
so \[ t \text{ is much lower than } 0.05 \]

Try \[ t = 0.01 \] (which makes annual yield 2%)
\[ P = 137.88 \]
Price too low, so yield too high.

Try \[ t = 0.005 \] (annual yield 1%)
then \[ P = 143.79 \] this is the closest
so annual yield is closest to 1% \( (A) \)
7. [4 marks]
Let \( X, Y \) and \( Z \) be matrices of sizes \( 2 \times 3 \), \( 4 \times 3 \) and \( 1 \times 4 \), respectively. Then the size of \( XY^T Z^T \) (where \( T \) denotes transpose of a matrix) is

A. the product is not defined
B. \( 2 \times 1 \)
C. \( 2 \times 4 \)
D. \( 4 \times 3 \)
E. \( 1 \times 2 \)

\[
\begin{align*}
    &XY^T Z^T \\
    &= 2\begin{pmatrix} 3 & 4 \\ 3 & 4 \end{pmatrix} \\
    &= 2 \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}
\end{align*}
\]

so multiplication is possible and the resulting size is \( 2 \times 1 \). \( \boxed{B} \)

8. [4 marks]
Which of the matrices below is not in completely reduced form?

A. \[
\begin{pmatrix}
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0
\end{pmatrix}
\]
B. \[
\begin{pmatrix}
1 & 0 & 2 \\
0 & 1 & -3
\end{pmatrix}
\]
C. \[
\begin{pmatrix}
0 & 1 & 0 & 2 \\
0 & 0 & 1 & 1
\end{pmatrix}
\]
D. \[
\begin{pmatrix}
1 & 2 & 3 & 4 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]
E. \[
\begin{pmatrix}
1 & 0 & 1 & 0 \\
0 & 0 & 1 & 1
\end{pmatrix}
\]

\( \boxed{E} \)
9. [4 marks]

When the matrix
\[
\begin{pmatrix}
2 & 3 \\
1 & 4 \\
0 & 1 \\
4 & 6 \\
6 & 2 \\
\end{pmatrix}
\]

is brought to reduced form, the number of zero rows is

A. none
B. 1
C. 2
D. 3
E. 4

Can change last 3 rows to zero rows by
\[
R_3 \rightarrow R_3 - 3R_2 - 2R_1,
R_4 \rightarrow R_4 - 6R_2 - 4R_1,
- R_5 \rightarrow R_5 - 2R_2 - 6R_1,
\]
so 3 zero rows. \(\boxed{D}\)

10. [4 marks]

Consider the system of equations
\[
x + y + 2z - w = 0 \\
3x - y + 4z + w = 0 \\
x - y + z + w = 0
\]

The system has
A. only the trivial solution
B. a one-parameter family of solutions
C. a two-parameter family of solutions
D. a three-parameter family of solutions
E. no solutions

\[
\begin{pmatrix}
1 & 1 & 2 & -1 \\
3 & -1 & 4 & 1 \\
1 & -1 & 1 & 1 \\
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 1 & 2 & -1 \\
0 & -4 & -2 & 4 \\
0 & -2 & -1 & 2 \\
\end{pmatrix}
\]

\[
R_3 \rightarrow \frac{1}{2}R_3, \\
R_3 \rightarrow 2R_3 + 2R_2
\]

Number of parameters = \(4 - 2 = 2\) \(\boxed{C}\)
PART B. Written-Answer Questions

1. [17 marks]
   A $15,000 loan at the interest rate of 9% per year compounded monthly is amortized over the next 10 years.

   [5] (a) Find the monthly payment of the loan (to the nearest cent). (You may assume payments begin at the end of the first month.)

   \[
   15,000 = R \frac{121.0075}{12}
   \]

   \[
   R = \frac{15,000 \times 0.0075}{1 - (1.0075)^{-120}} = \$190.01
   \]

   For items (b) and (c), assume that after 5 years, the interest rate changes to 12% compounded monthly.

   [6] (b) If the entire loan must still be paid off in 10 years, find the new monthly payment for the last 5 years (to the nearest cent).

   \[
   \text{Principal outstanding} = R \frac{121.0075}{60} \text{ with } R \text{ from (a)}
   \]

   \[
   = \$9153.60 = T \frac{121.01}{60} \text{ (T is new payment)}
   \]

   \[
   T = \frac{9153.60 \times 0.01}{1 - (1.01)^{-60}} \approx \$203.62
   \]

   [6] (c) If, instead of (b), the payment remains the same as in (a), how many more full payments would have to be made to repay the loan?

   \[
   \frac{9153.60}{190.01} = a_{66.01}
   \]

   \[
   \frac{9153.60}{190.01} = 1 - (1.01)^{-n}
   \]

   \[
   1 - \frac{0.01 \times 9153.60}{190.01} = (1.01)^{-n}
   \]

   \[
   - \ln \left(1 - \frac{0.01 \times 9153.60}{190.01}\right) = n
   \]

   \[
   \ln (1.01)
   \]

   \[
   n \approx 66.06 \text{ There would be 66 full payments}
   \]

   i.e. [6 more] than in the arrangement in (a)
2. [16 marks]

A $500,000 25-year mortgage has weekly payments and an interest rate of 6% compounded semi-annually.

[Assume that every year has 52 weeks exactly.]

(a) Find the weekly payment (to the nearest cent).

If \( i \) is weekly interest:

\[
(1 + i)^{52} = (1.03)^2
\]

\[
R = \frac{500,000i}{1 - (1+i)^{-1300}} = \frac{500,000 \left[ (1.03)^{26} - 1 \right]}{1 - (1.03)^{-30}}
\]

\[
R = 736.84
\]

(b) How much total interest would be saved if the amortization period were reduced to 15 years?

If amortization were reduced to 15 yrs, payments \( T \),

\[
500,000 = \frac{T a_{\overline{15\times 52}}}{i}
\]

\[
T = \frac{500,000i}{1 - (1+i)^{-780}} = \frac{500 \left[ (1.03)^{26} - 1 \right]}{1 - (1.03)^{-30}}
\]

\[
T = 967.26
\]

In (a) Total Interest is: \( 1300 \times 736.84 - 500,000 \)

In (b) Total Interest is: \( 780 \times 967.26 - 500,000 \)

Taking the difference (or noticing directly that since the original amount of the mortgage is the same, the difference in total payments must equal the difference in interest payments)

Total Interest saved = \( 1300 \times 736.84 - 780 \times 967.26 \)

\[
\approx 203,429.20
\]
3. **[13 marks]**

   Use the method of row reduction to find all solutions of the system
   \[
   \begin{align*}
   2x + 3y + 12z &= 4 \\
   3x - 2y + 5z &= 5 \\
   4x + y + 14z &= 6
   \end{align*}
   \]

   **[No marks will be given for any method other than row reduction.]**

   \[
   \begin{pmatrix}
   2 & 3 & 12 \\
   3 & -2 & 5 \\
   4 & 1 & 14
   \end{pmatrix}
   \xrightarrow{R_1 \rightarrow \frac{1}{2} R_1}
   \begin{pmatrix}
   1 & \frac{3}{2} & 6 \\
   3 & -2 & 5 \\
   4 & 1 & 14
   \end{pmatrix}
   \xrightarrow{R_2 \rightarrow R_1 - 3R_1}
   \begin{pmatrix}
   1 & \frac{3}{2} & 6 \\
   0 & -\frac{1}{2} & -13 \\
   4 & 1 & 14
   \end{pmatrix}
   \]

   \[
   \xrightarrow{R_3 \rightarrow R_3 - 4R_1}
   \begin{pmatrix}
   1 & \frac{3}{2} & 6 \\
   0 & -\frac{1}{2} & -13 \\
   0 & -5 & -10
   \end{pmatrix}
   \]

   \[
   \xrightarrow{R_2 \rightarrow -\frac{2}{5} R_2}
   \begin{pmatrix}
   1 & \frac{3}{2} & 6 \\
   0 & 1 & 2 \\
   0 & -5 & -10
   \end{pmatrix}
   \]

   \[
   \xrightarrow{R_3 \rightarrow R_3 + 5R_2}
   \begin{pmatrix}
   1 & \frac{3}{2} & 6 \\
   0 & 1 & 2 \\
   0 & 0 & 0
   \end{pmatrix}
   \]

   Since \[\frac{10}{13} - 2 \neq 0,\] **There are no solutions.**
4. [14 marks]

A furniture manufacturer produces two types of chairs, A and B. Chair A requires 7 nails and 4 brackets. Chair B requires 3 nails and 5 brackets. How many of each type of chair can be produced if 92 nails and 69 brackets are available and all must be used?

[Solve the linear equations using the method of row reduction. Many marks will be deducted for any other method of solution.]

\[
\begin{align*}
7A + 3B &= 92 \\
4A + 5B &= 69
\end{align*}
\]

\[
\begin{pmatrix}
7 & 3 & | & 92 \\
4 & 5 & | & 69
\end{pmatrix} \rightarrow
\begin{pmatrix}
1 & \frac{3}{7} & | & \frac{92}{7} \\
0 & \frac{23}{7} & | & \frac{115}{7}
\end{pmatrix}
\]

\[
R_2 \rightarrow R_2 - 4R_1
\]

\[
\begin{pmatrix}
1 & \frac{3}{7} & | & \frac{92}{7} \\
0 & 1 & | & \frac{15}{7}
\end{pmatrix}
\]

So, \( B = 5 \) and \( A = \frac{92}{7} - \frac{3}{7} B = \frac{92 - 15}{7} = \frac{77}{7} = 11 \) OK

\[
R_1 \rightarrow R_1 - \frac{3}{7} R_2
\]

\[
\begin{pmatrix}
1 & 0 & | & 11 \\
0 & 1 & | & 5
\end{pmatrix}
\]

In either case, \( A = 11 \) and \( B = 5 \).