Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 10 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor's name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it. ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME:

GIVEN NAME:

STUDENT NO:

SIGNATURE:

TUTORIAL TIME and ROOM:

REGCODE and TIMECODE:

T.A.'S NAME:

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FOR MARKER ONLY

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PART A. Multiple Choice

1. [4 marks]
   If the rate of interest is 5% compounded continuously, then, in 10 years, $100,000 will accumulate to
   \[ 100,000e^{0.05 \times 10} = \]
   \[ \$164,872 \]
   A. $164,872
   B. $162,889
   C. $150,000
   D. $137,111
   E. $105,127

2. [4 marks]
   **Hint:** Note that 52 weeks = 1 year = 12 months.
   The effective weekly rate of interest most nearly equivalent to an effective monthly rate of 1% is
   \[ (1.01)^{12} = (1 + t)^{52} \]
   \[ t = (1.01)^{12/52} - 1 \]
   A. 0.2182%
   B. 0.2308%
   C. 0.2351%
   D. 0.2439%
   E. 0.2299%
3. [4 marks]

10 annual deposits of $1,000 each are made into an account earning 5% compounded annually. 6 years after the last deposit, the account will amount to

A. $13,206.79
B. $16,855.58
C. $13,400.96
D. $12,577.89
E. $16,288.95

\[
1000 \left( \frac{1}{1.05} \right)^6 = 1000 \left[ \frac{(1.05)^6 - 1}{0.05} \right] (1.05)^6
\]

\[
= 1000 \left( \frac{1.05^6 - 1}{0.05} \right) (1.05)^6
\]

\[
= \$16,288.95
\]

4. [4 marks]

If a $500,000 mortgage is amortized over 25 years at 6% compounded semi-annually with monthly payments, then each payment is closest to

A. $3,199.03
B. $5,042.08
C. $15,002.11
D. $28,713.94
E. $39,113.36

\[
(1.03)^2 = (1 + 0.01)^{12}
\]

\[
500,000 = RA \frac{1}{300i}
\]

\[
R = \frac{500,000 \cdot i}{1 - (1+i)^{-300}}
\]

\[
R = \frac{500,000 \left[ (1.03)^{15} - 1 \right]}{1 - (1.03)^{-300}}
\]

\[
R \approx \$199.03
\]
5. [4 marks]

If a $100 bond has 7 years until maturity, semi-annual coupons worth $4 each, and an annual yield to maturity of 8%, then its market price is

A. $67.02
B. $79.17
C. $92.54
D. $93.95
E. $100.00

Alternatively:

\[ P = 100(1.04)^{-14} + 40\frac{1}{1.04^{14}} \]

\[ = $100 \]

6. [4 marks]

If a $100 bond with semi-annual coupons has 9 years until maturity, an annual yield to maturity of 7%, and sells for $90, then each semi-annual coupon is closest to

A. $2.24
B. $2.62
C. $2.74
D. $3.49
E. $3.63

\[ 90 = 100(1.035)^{-18} + (100r)\frac{1}{1.035^{18}} \]

\[ 100r = \frac{90 - 100(1.035)^{-18}}{1.035^{18}} \]

\[ 100r = 2.74 \]
7. [4 marks]

Let \( X = \begin{pmatrix} 1 & -1 & 0 & 4 & 5 \\ 0 & 2 & 3 & 7 & 1 \end{pmatrix}, \quad Y = \begin{pmatrix} 4 & 5 \\ 6 & 7 \end{pmatrix} \).

Which one of the following is not defined?

A. \( Y^T X \)
B. \( Y^T Y X \)
C. \( YXY \)
D. \( (Y + Y^T) X \)
E. \( YY^T YY^T \)

\[ \begin{array}{c}
A. 2^{t+4} Y^T 2^X \text{ OK} \\
B. 2^{t+4} Y^T 2^X 2^Y \text{ OK} \\
C. 2^{t+4} Y^T 2^X 2^Y \text{ OK} \\
D. 2^{t+4} (Y + Y^T) 2^X \text{ OK} \\
E. 2^{t+4} Y^T 2^Y 2^Y \text{ OK} \\
\end{array} \]

8. [4 marks]

When the matrix \( \begin{pmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 3 & 0 & 2 \\ 2 & -1 & 2 \end{pmatrix} \) is reduced, the number of non-zero rows is

A. 0
B. 1
C. 2
D. 3
E. 4

\[ \begin{array}{c}
\begin{array}{c}
R_2 \leftrightarrow R_1 \\
R_3 \rightarrow R_3 - 3R_1 \\
R_4 \rightarrow R_4 - 2R_1 \\
R_4 \leftrightarrow R_2 \\
R_3 \rightarrow R_3 - 3R_2 \\
R_4 \rightarrow R_4 - 3R_2 \\
\end{array} \\
\begin{pmatrix} 1 & -1 & 1 \\ 0 & 3 & -1 \\ 0 & 1 & 0 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \\
\end{array} \]

3 non-zero rows
9. [4 marks]
Consider the system
\[\begin{align*}
2x - y + 3z - 4w + 5v &= 0 \\
x + 2y + 2z + w - v &= 0
\end{align*}\]
The system has
A. only the trivial solution \(x = y = z = w = v = 0\)
B. a one-parameter family of solutions
C. a two-parameter family of solutions
\[\begin{vmatrix} 1 & 2 & 2 & 1 & -1 & |0 \\ 2 & -1 & 3 & -4 & 5 & |0 \end{vmatrix}\]
D. a three-parameter family of solutions
E. a four-parameter family of solutions
\[\begin{vmatrix} 1 & 2 & 2 & 1 & -1 & |0 \\ 0 & -5 & -1 & -6 & -7 & |0 \end{vmatrix}\]

There will be 2 non-zero rows and 5 variables, hence \(5 - 2 = 3\) parameters

10. [4 marks]
If \(A^{-1} = \begin{bmatrix} -1 & 2 & 2 \\ 1 & -3 & -2 \\ 1 & -1 & -1 \end{bmatrix}\) \(X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}\) \(C = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\)

then the solution to the matrix equation \(AX = C\) is given by
A. \(x = 1\) \(y = -3\) \(z = -2\)
B. \(x = -1\) \(y = -3\) \(z = 3\)
C. \(x = 3\) \(y = -3\) \(z = 0\)
D. \(x = 0\) \(y = -3\) \(z = -1\)
E. \(x = 1\) \(y = -3\) \(z = 0\)

\[\begin{align*}
X &= A^{-1}C \\
\begin{pmatrix} -1 & 2 & 2 \\ 1 & -3 & -2 \\ 1 & -1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 + 4 - 2 \\ 1 - 6 + 2 \\ 1 - 2 + 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -3 \end{pmatrix}
\end{align*}\]

\(X = 1\) \(Y = -3\) \(Z = 0\)
PART B. Written-Answer Questions

1. [15 marks]

Ms. Clark wishes to accumulate $10,000 by depositing $100 each month into an account earning 3.6% per year compounded monthly.

(a) How many $100 deposits must she make before the account has more than $9,900?

\[ 9900 = 100 \left( \frac{1}{1.003} \right)^n - 1 \]

\[ 99 \times 1.003 + 1 = (1.003)^n \]

\[ 1.297 = (1.003)^n \]

\[ \ln(1.297) = n \ln(1.003) \]

\[ n = \frac{\ln(1.297)}{\ln(1.003)} \approx 86.8 \quad \text{so 86 not quite enough, but by 87 OK} \]

(b) Let \( n \) denote the answer to part (a). One month after her \( n \)th $100 deposit, Ms. Clark makes a last deposit to bring her account to exactly $10,000. How much is this last deposit?

\[ \frac{37}{100} \quad \frac{88}{100} \quad x = 10,000 \]

\[ 100 \times \frac{37}{100} + x = 10,000 \]

\[ x = \$ 46,21 \]
2. [15 marks]
A person puts a down-payment of $70,000 on a house and sets up a mortgage to pay for the rest. The mortgage is amortized over 20 years at 6.2% per annum compounded semi-annually, with monthly payments of $915.00.

7/ (a) How much was the house worth at the start?

\[
(1 + i)^{12} = (1.031)^2
\]

\[
A = 70,000 + 915 \frac{A}{2401} i
\]

\[
= 70,000 + 915 \left[ 1 - (1 + i)^{-240} \right] \frac{i}{i} = 915 \left[ 1 - (1.031)^{-40} \right] \frac{(1.031)^{21} - 1}{(1.031)^{21} - 1} + 70,000
\]

\[
A = \$196,477 \text{ to the nearest dollar}
\]

8/ (b) How much more interest is contained in the 14th payment than in the 20th payment of the mortgage?

P.O. at beginning of 20th period = $915A \frac{2271}{2271} i$

Interest in 20th payment = $i \times 915A \frac{2271}{2271} i$

Similarly, interest in 14th payment = $i \times 915A \frac{2271}{2271} i$

Interest in 14th - Interest in 20th

\[
= 915 i \left[ A \frac{2271}{2271} i - A \frac{2271}{2271} i \right]
\]

\[
= 915 \left[ (1 - (1 + i)^{-227}) - (1 - (1 + i)^{-221}) \right]
\]

\[
= 915 \left[ (1 + i)^{-221} - (1 + i)^{-227} \right]
\]

\[
= 915 \left[ (1.031)^{-221/6} - (1.031)^{-227/6} \right]
\]

\[
= \$8.94
\]
3. [15 marks]

Use the method of reduction of matrices to find all solutions of

\[
\begin{align*}
x + y + 4z &= 1 \\
2x - 3y + 3z &= -8 \\
3x + 2y + 11z &= 1 \\
4x + 2y + 14z &= 0
\end{align*}
\]

[No other method will be given any marks.]

\[
\begin{pmatrix}
1 & 1 & 4 & 1 \\
2 & -3 & 3 & -8 \\
3 & 2 & 11 & 1 \\
4 & 2 & 14 & 0
\end{pmatrix}
\]

\[
\begin{align*}
R_2 &\rightarrow R_2 - 2R_1 \\
R_3 &\rightarrow R_3 - 3R_1 \\
R_4 &\rightarrow R_4 - 4R_1 \\
R_2 &\rightarrow \frac{1}{2}R_2 \\
R_3 &\rightarrow R_3 + R_2 \\
R_4 &\rightarrow R_4 + 2R_2
\end{align*}
\]

\[
\begin{pmatrix}
1 & 1 & 4 \\
0 & 1 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 1 & 4 & 1 \\
0 & 1 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 3 & -1 \\
0 & 1 & 1 & 2 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

\[
X = -1 - 3Z \\
Y = 2 - Z
\]

or

\[
X = -1 - 3r \\
Y = 2 - r \\
Z = r
\]
4. [15 marks]

An insurance company has three types of documents to process (types A, B and C). The following table shows how many hours each document needs with each of the accountant, lawyer and secretary.

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<th>Lawyer</th>
<th>Secretary</th>
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<td>3</td>
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<tr>
<td>B</td>
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<td>2</td>
<td>3</td>
</tr>
<tr>
<td>C</td>
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The accountant has a total of 34 hours, the lawyer has 35 hours, and the secretary has 36 hours to spend. How many documents of each type can they process?

Let \( A, B, C \) be the number of documents of types A, B and C respectively.

\[
\begin{align*}
2A + 4B + 2C &= 34 \\
3A + 2B + 4C &= 35 \\
3A + 3B + 3C &= 36
\end{align*}
\]

![Matrix and operations diagram]

\[
\begin{align*}
\begin{bmatrix} 2 & 4 & 2 \\ 3 & 2 & 4 \\ 3 & 3 & 3 \end{bmatrix} &\xrightarrow{R_1 \rightarrow \frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & 3 & 0 \end{bmatrix} \\
&\xrightarrow{R_2 \rightarrow 3R_1 - 2R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\
&\xrightarrow{R_3 \rightarrow R_3 + 2R_2} \begin{bmatrix} 1 & 2 & 1 \\ 0 & 4 & 1 \\ 0 & 0 & 1 \end{bmatrix} \\
\end{align*}
\]

So \( C = 4, B = 5 \) and \( A = 17 - C - 2B = 17 - 4 - 10 = 3 \)

Type A = 3, Type B = 5, Type C = 4

Alternatively, proceeding to complete reduction

\[
\begin{align*}
R_1 &\rightarrow R_1 - R_3 - 2R_2 \\
&\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 5 \\ 4 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}
\end{align*}
\]

yielding the same answer