Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 10 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor’s name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it. ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME: ________________________________

GIVEN NAME: ________________________________

STUDENT NO: ________________________________

SIGNATURE: ________________________________

TUTORIAL TIME and ROOM: ________________________________

REGCODE and TIMECODE: ________________________________

T.A.’S NAME: ________________________________

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FOR MARKER ONLY

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PART A. Multiple Choice

1. [4 marks]
What nominal annual rate compounded weekly is most nearly equivalent to 5% per year compounded quarterly? [For this question, 1 year = 52 weeks.]

A. 4.89%
B. 4.97%
C. 5.09%
D. 5.00%
E. 4.93%

Let \( r = \text{nominal annual rate} \)

\[
(1 + \frac{r}{52})^{52} = (1 + \frac{.05}{4})^4
\]

\[
r = 52 \left[ \left( 1 + \frac{.05}{4} \right)^{\frac{1}{13}} - 1 \right]
\]

\[
r = .04971
\]

2. [4 marks]
A loan of $10,000 is to be repaid with payments of $X one year from now and $2X three years from now. If the effective annual rate of the loan is 10%, then \( X = \)

A. 4146.42
B. 3908.54
C. 3769.47
D. 4561.06
E. 5017.17

\[
0 \quad 1 \quad 3
\]

\[
\begin{array}{ccc}
10,000 & X & 2X \\
\end{array}
\]

\[
10,000 = X (1.10)^{-1} + 2X (1.10)^{-3}
\]

\[
X = \frac{10,000}{(1.10)^{-1} + 2(1.10)^{-3}}
\]

\[
X = 4146.417
\]
3. [4 marks]
If interest is compounded continuously, what annual rate (to the nearest 0.01%) is required if the amount in an account is to double in 8 years?

A. 12.50%
B. 7.92%
C. 9.05%
D. 10.63%
E. 8.66%

\[ P = P_0 e^{rt} \]
\[ 2P_0 = P_0 e^{8r} \]
\[ \ln 2 = 8r \]
\[ r = \frac{1}{8} \ln 2 \]
\[ = 0.08664 \ldots \]

4. [4 marks]
A father deposits $1,500 in an account on the day of his son's birth and continues to make similar deposits every year on his son's birthday up to and including his 17th birthday. If the account earns an effective annual rate of 5% then how much will there be in the account on his 17th birthday just after that day's deposit?

A. $40,698.58
B. $17,534.38
C. $42,198.58
D. $38,760.55
E. $37,260.55

Note: There are 18 payments.

\[ 1500 \times \frac{(1.05)^{18} - 1}{0.05} \]
\[ = 42,198.577 \]

Other ways, e.g.
\[ 1500 (1.05)^{17} + 1500 \times \frac{(1.05)^{18} - 1}{0.05} \]

will get the same answer.
5. [4 marks]
How many semi-annual interest payments are remaining for a $100 bond with annual coupon rate of 4% and annual yield rate very close to 4.5% if the bond is selling for $96 per $100 of face value and the next interest payment is in 6 months?

\[
q_6 = 100 \left(1.0225\right)^{-n} + 2 \left(0.0225\right) \ln \frac{1}{1.0225} \]

\[
96 = 100 \left(1.0225\right)^{-n} + 2 \left[1 - \left(1.0225\right)^{-n}\right]
\]

B) \hspace{1cm} \text{Solve for } \left(1.0225\right)^{-n}:

\[
96 \times 0.0225 - 2 = (2.25 - 2) \left(1.0225\right)^{-n}
\]

\[
\ln \frac{16}{25} = -n \ln 1.0225
\]

\[
n = \frac{-\ln \frac{16}{25}}{-\ln 1.0225} = 20.057
\]

6. [4 marks]
To purchase a $450,000 house a person pays $50,000 down and takes on a 25 year mortgage with monthly payments and interest at 6% compounded semi-annually. The monthly mortgage payments will be closest to:

\[
\frac{450,000 - 50,000}{300} \cdot (1+i)^{12} = (1.03)^2
\]

\[
R = \frac{400,000 \cdot i}{1 - (1+i)^{-300}} = \frac{400,000 \left[(1.03)^{12} - 1\right]}{1 - (1.03)^{-300}} = 2559.23
\]
7. [4 marks]
A 20 year loan for $100,000 is to be amortized by equal semi-annual payments. If interest is at the nominal rate of 10% per year compounded semi-annually, then the semi-annual payments are $5,827.82 (You can check this if you have time to waste.) The interest in the 21st payment is closest to

\[ \text{P.O. after 20th payment is} \]

\[ 5827.82 \left[ 1 - (1.05)^{-20} \right] \]

\[ .05 \]

\[ \text{because there are 20 payments remaining:} \]

\[ \text{Interest in the next payment is} \]

\[ .05 \times \text{P.O.} \]

\[ = 5827.82 \left[ 1 - (1.05)^{-20} \right] \]

\[ = 3631.37 \ldots \]

8. [4 marks]
Let \( A = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 1 & 3 \end{pmatrix} \)
\( B = \begin{pmatrix} 2 & 1 \\ -1 & 1 \end{pmatrix} \)
\( C = \begin{pmatrix} 1 & 3 \\ 4 & 4 \end{pmatrix} \)

Which (one) of the following products exists?

A. \( C^{-1}B \)
B. \( C^{T}AB \)
C. \( BBT^{A} \)
D. \( A^{-1}B \)
E. \( BCB^{-1} \)

A. \( C^{-1} \) cannot exist
B. \( C^{T} \) cannot multiply \( A \)
C. \( 2B \) and \( 2A \) exists
D. \( A^{-1} \) cannot exist
E. \( 2B + 3C \) cannot be multiplied
9. [4 marks]

The system of equations
\[
\begin{align*}
x + y + 2z &= 1 \\
x - 5z &= -1 \\
3x + 2y - z &= 2
\end{align*}
\]

has

A. the complete solution \( x = -1, y = 2, z = 0 \)
B. the complete solution \( x = 5z - 1, y = 2 - 7z, z \) any real number
C. the complete solution \( x = 4, y = -5, z = 1 \)
D. the complete solution \( x = (3 - 5y)/7, y \) any real number, \( z = (2 - y)/7 \)

E. no solution

\[
\begin{pmatrix}
1 & 1 & 2 \\
1 & 0 & -5 \\
3 & 2 & -1
\end{pmatrix}
\begin{pmatrix}
R_2 \rightarrow R_2 - R_1 \\
R_3 \rightarrow R_3 - 3R_1 \\
R_2 \rightarrow -R_2 \\
R_3 \rightarrow R_3 + R_2
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 2 & \mid & 1 \\
0 & -1 & -7 & \mid & -2 \\
0 & 1 & 7 & \mid & 2
\end{pmatrix}
\]

no solution

10. [4 marks]

The \( a_{32} \) entry in the inverse of the matrix
\[
\begin{pmatrix}
1 & 0 & 1 \\
1 & -1 & 0 \\
0 & 2 & 1
\end{pmatrix}
\]
is

A. 1
B. -2
C. 0
D. -1
E. 2

\[
\begin{pmatrix}
1 & 0 & 1 \\
1 & -1 & 0 \\
0 & 2 & 1
\end{pmatrix}
\begin{pmatrix}
R_2 \rightarrow R_2 - R_1 \\
R_3 \rightarrow R_3 - 2R_2 \\
R_3 \rightarrow R_3 - 2R_2
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 1 & \mid & 1 & 0 & 0 \\
0 & 1 & 1 & \mid & 1 & -1 & 0 \\
0 & 0 & 1 & \mid & -2 & 2 & 1
\end{pmatrix}
\]

\( a_{32} = -2 \)
1. [15 marks]
A pile of coins consists of nickels, dimes, and quarters. There are 18 coins in the pile. The total value is $2.55. The number of quarters is one more than the number of nickels. How many nickels, dimes, and quarters are in the pile?

[For full marks, make sure you use row-reduction to solve the system of equations.]

Let \( N = \# \text{ nickels} \)
\( D = \# \text{ dimes} \)
\( Q = \# \text{ quarters} \)

\[
\begin{align*}
N + D + Q &= 18 \\
5N + 10D + 25Q &= 255
\end{align*}
\]

\[
\begin{array}{c}
N + D + Q = 18 \\
N + 2D + 5Q = 51 \\
Q = N + 1
\end{array}
\]

\[
\begin{pmatrix}
1 & 1 & 1 & 18 \\
1 & 2 & 5 & 51 \\
-1 & 0 & 1 & 1
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 1 & 1 & 18 \\
0 & 1 & 4 & 33 \\
0 & 0 & 2 & -14
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 1 & 1 & 18 \\
0 & 1 & 4 & 33 \\
0 & 0 & 1 & 7
\end{pmatrix}
\]

\( Q = 7 \)
\( D = 33 - 4Q = 5 \)
\( N = 18 - Q - D = 6 \)

6 nickels, 5 dimes, 7 quarters

Alternatively, substitute \( Q = N + 1 \) into last two equations:

\[
\begin{align*}
N + D + N + 1 &= 18 \\
5N + 10D + 25(N + 1) &= 255
\end{align*}
\]

\[
\begin{align*}
2N + D &= 17 \\
3N + D &= 23
\end{align*}
\]

Better still:

\[
\begin{align*}
D + 2N &= 17 \\
D + 3N &= 23
\end{align*}
\]

\[
\begin{pmatrix}
1 & 2 & 17 \\
0 & 1 & 6
\end{pmatrix}
\rightarrow
\begin{pmatrix}
1 & 2 & 17 \\
0 & 1 & 6
\end{pmatrix}
\]

So \( N = 6 \), \( D = 17 - 2N = 5 \) and \( Q = N + 1 = 7 \) as before

or \( \begin{pmatrix} 1 & 0 & 5 \\ 0 & 1 & 6 \end{pmatrix} \) same answer $600$.  

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2. [16 marks]

The input/output matrix for the industries A and B is given by:

\[
\begin{pmatrix}
\text{Industry A} & \text{Industry B} & \text{Final Demand} \\
100 & 300 & 100 \\
300 & 225 & 225 \\
100 & 225 & -
\end{pmatrix}
\]

If the final demand changes to 160 for Industry A and to 200 for Industry B then find:

\[\text{Total output A} = 500 \quad \text{Total output B} = 750\]

\[
\text{Tech matrix} = \begin{pmatrix}
\frac{4}{5} & \frac{2}{5} \\
\frac{2}{5} & \frac{3}{10} \\
\frac{3}{10} & \frac{3}{10}
\end{pmatrix}
\]

\[
\text{Lambert}^{\text{F}} = \begin{pmatrix}
\frac{4}{5} & -\frac{2}{5} \\
-\frac{3}{5} & \frac{7}{10}
\end{pmatrix}
\]

\[
\begin{align*}
\frac{4}{5}A - \frac{2}{5}B &= 160 \\
-\frac{3}{5}A + \frac{7}{10}B &= 200
\end{align*}
\]

\[
\begin{pmatrix}
\frac{4}{5} & -\frac{2}{5} & 160 \\
-\frac{3}{5} & \frac{7}{10} & 200
\end{pmatrix}
\]

\[
\begin{pmatrix}
R_1 & -\frac{2}{5}R_1 & 160 \\
-\frac{3}{5}R_2 & \frac{7}{10}R_2 & -1 & \frac{7}{6} & 200
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & -\frac{1}{2} & 200 \\
0 & \frac{3}{5} & \frac{1600}{3}
\end{pmatrix}
\]

\[
\begin{pmatrix}
R_2 \to R_2 + \frac{2}{3}R_1 \\
1 & 0 & 600
\end{pmatrix}
\]

\[
\begin{pmatrix}
1 & 0 & 800 \\
0 & 1 & 800
\end{pmatrix}
\]

\[
A = 600 \quad \text{and} \quad B = 800
\]

\[4/ \text{(b)} \quad \text{The new other production factors for industries A and B.} \]

\[
\begin{align*}
\text{For industry A} & \quad \frac{1}{5} \times 600 = 120 \\
\text{For industry B} & \quad \frac{3}{10} \times 800 = 240
\end{align*}
\]
3. [14 marks]

Veronica has won a lottery which will pay her $3,000 each month for 1 year, and then $1,000 each month for the next 3 years. She will receive her first $1,000 payment 1 month after her last $3,000 payment. If she invests each lottery cheque just after receiving it, at 4.5% compounded monthly, what will be the amount of her investment immediately after she deposits her last lottery payment?

We need the value at 48 of all these payments using \( i = \frac{0.045}{12} = 0.00375 \)

Could do: \( (1) \) \( 3000 \times 48 \times i - 2000 \times \frac{36i}{i} \)

\( \text{or } (2) 3000 \times \frac{36i}{12i} (1+i)^{36} + 1000 \times \frac{36i}{i} \)

\( \text{or } (3) 3000 \times \frac{36i}{12i} + 1000 \times \frac{36i}{i} (1+i)^{36} \)

All of these come to \( \$80,519.32 \)

Explanation:

1. An ordinary annuity of \( \$3000 \) for 48 periods take away (to make \( $1000 \)) an ordinary annuity of \( $2000 \) for 36 periods.

2. Accumulate the 12 \( $3000 \) payments and send them as a lump to the end, then add the annuity of 36 \( $1000 \) payments

3. Accumulate all payments to the moment marked "12" on the time line, then send as a lump to the end.
4. [15 marks]

A bond with an 8% annual coupon rate and semi-annual coupons has 25 coupons remaining with the first one cashable 6 months from now.

[10] (a) If the bond is currently selling at $120 per $100 of face value, find the annual yield to maturity. [The price should be correct to within $1 of the actual price per $100 of face value.]

\[ 120 = 100 \left(1 + \frac{r}{2}\right)^{-25} + 4 \cdot \frac{1}{2} \cdot 100 \cdot r < 0.04 \]

\[ r = 0.03 \Rightarrow P = 117.41 \text{ too low; yield too high} \]

\[ r = 0.025 \Rightarrow P = 127.64 \text{ too high; yield too low} \]

\[ r = 0.029 \Rightarrow P = 119.37 \text{ this is within $1 of 120} \]

\[ \text{Yield} = 5.8\% \]

5.6\% or 5.9\% are not acceptable since they give 121.37 and 118.39.

Any yield to maturity greater than 5.6\% and less than 5.9\% will do. \(5.7\%\), for example, gives $120.36 which is better than the given answer.

[5] (b) On the same day as in part (a), the issuers of the bond announce that those who so choose will be able, in 5 years time right after the coupon payment, to exchange each $100 of face value with its remaining coupons for $105 in cash. If the annual yield to maturity remains the same as in (a), what happens to the price of the bond? [Numerical calculation, with explanation please]

The issuers are offering $105 in cash 5 yrs. from now plus 10 coupon payments of $4 each.

The present value of this offer is

Using 5.8\%, \(105 \left(1.029\right)^{-10} + 4 \cdot 101.029 = 113.19\)

Using 5.7\%, \(105 \left(1.0285\right)^{-10} + 4 \cdot 101.0285 = 113.66\)

In either case, nothing happens to the price of the bond, because no one will plan to exchange a bond worth 120.36 for one worth 113 + pennies, and since they don't have to, they won't.