Aids Allowed: A non-graphing calculator, with empty memory, to be supplied by student.

Instructions: Fill in the information on this page, and make sure your test booklet contains 12 pages. In addition, you should have a multiple-choice answer sheet, on which you should fill in your name, number, tutorial time, tutorial room, and tutor's name.

This test consists of 10 multiple choice questions, and 4 written-answer questions. For the multiple choice questions you can do your rough work in the test booklet, but you must record your answer by circling the appropriate letter on the answer sheet with your pencil. Each correct answer is worth 4 marks; a question left blank, or an incorrect answer, or two answers for the same question is worth 0. For the written-answer questions, present your solutions in the space provided. The value of each written-answer question is indicated beside it. ENCLOSE YOUR FINAL ANSWER IN A BOX AND WRITE IT IN INK.

TOTAL MARKS: 100

FAMILY NAME: ______________________________________________________

GIVEN NAME: ______________________________________________________

STUDENT NO: ______________________________________________________

SIGNATURE: ________________________________________________________

TUTORIAL TIME and ROOM: ____________________________________________

REGCODE and TIMECODE: ____________________________________________

T.A.'S NAME: ________________________________________________________

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FOR MARKER ONLY

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PART A. Multiple Choice

1. [4 marks]

In the solution to the following system

\[
\begin{align*}
2x + y - z &= 7 \\
x + 3y + z &= 4 \\
3x + y + z &= 2
\end{align*}
\]

\[
z = \begin{vmatrix}
2 & 1 & 7 \\
1 & 3 & 4 \\
3 & 1 & 2 \\
\end{vmatrix} = \begin{vmatrix}
0 & -5 & -1 \\
1 & 3 & 4 \\
0 & -8 & -10 \\
\end{vmatrix} = \begin{vmatrix}
-5 & -1 \\
-8 & -10 \\
\end{vmatrix}
\]

\[
= \frac{50 - 8}{10 - 24} = \frac{42}{-14} = -3
\]

2. [4 marks]

Assume \( p(x) \) and \( q(x) \) are polynomials and that \( \lim_{x \to -1} \frac{p(x)}{q(x)} \) exists and is equal to some nonzero value \( c \). Then

\[
\lim_{x \to -1} \frac{q(x)^2 + p(x) q(x)}{p(x)^2} = \lim_{x \to -1} \left[ \frac{q(x)}{p(x)} \right]^2 + \lim_{x \to -1} \left[ \frac{q(x)}{p(x)} \right]\]

\[
= \frac{1}{c^2} + \frac{1}{c}
\]

A. \( c^2 + c \)
B. \( c^{-2} + c^{-1} \)
C. \( c^2 + c^{-1} \)
D. \( c^{-2} + c \)
E. cannot be determined from the given information.
3. \(4\) marks
\[
\lim_{x \to \infty} \frac{(2x + 1)(x - 3)}{8x^2 - 5x + 2} = \lim_{x \to \infty} \frac{(2 - \frac{1}{x})(1 - \frac{3}{x})}{8 - \frac{5}{x} + \frac{2}{x^2}} = \frac{2}{8} = \frac{1}{4}
\]
A. Undefined
B. \(\infty\)
C. \(-3/2\)
D. 0
E. \(1/4\)

4. \(4\) marks
\[
\lim_{x \to -1^-} 5 - 3\frac{1}{1-x^2} = \frac{x}{-1} \quad x \leq -1
\]
\[x^2 > 1 \quad \text{as} \quad x \to -1^- \]
\[1 - x^2 < 0 \quad \frac{1}{1-x^2} \to -\infty \quad \text{as} \quad x \to -1^- \]
\[3 \frac{x}{1-x^2} \to 0 \quad \text{as} \quad x \to -1^- \]
\[
\lim_{x \to -1^-} 5 - 3\frac{1}{1-x^2} = 5
\]
5. [4 marks]
\[
\frac{(x^2 - 1)^2}{x} > 0 \text{ precisely when}
\]
A. \(-\infty < x < -1\) or \(0 < x < 1\)
B. \(-1 < x < 0\) or \(1 < x < \infty\)
C. \(-1 < x < 1\)
D. \(x < -1\) or \(x > 1\)
E. \(x > 0\) and \(x \neq 1\)

\[
\frac{(x^2 - 1)^2}{x} > 0 \implies x \neq 0 \quad \text{and} \quad x \neq 1
\]

6. [4 marks]
If \(y = (1-x)(2-x)(3-x)^3\) then \(y'(0) = \)

A. 108
B. -243
C. -324
D. -81
E. -108

\[
y'(0) = 4\pi^2 \quad \text{or} \quad \begin{bmatrix}
-1 \\
-1 \\
-1
\end{bmatrix} = \begin{bmatrix}
-324
\end{bmatrix}
\]

\[
y' = -(2-x)^3(3-x)^3 - 2(1-x)(2-x)(3-x)^3 - 3(1-x)(2-x)^2
\]

\[
y'(0) = -4.27 - 2.27 - 3.49 = -324
\]
7. [4 marks]

The slope of the tangent to the graph of \( y = \sqrt{2e^x + \sqrt{3e^x + e^{-x}}} \) at the point \((0, 2)\) is

A. \(\frac{3}{4}\)
B. \(\frac{5}{8}\)
C. 0
D. 2
E. \(\frac{5}{2}\)

\[
\begin{align*}
\frac{dy}{dx} &= \frac{1}{2 \sqrt{2e^x + 3e^x + e^{-x}}} \left( 2e^x + \frac{3e^x - e^{-x}}{2 \sqrt{3e^x - e^{-x}}} \right) \\
&= \frac{1}{2 \sqrt{2 + \frac{3}{4}}} \left( 2 + \frac{2}{2 \sqrt{4}} \right) \\
&= \frac{1}{2 \cdot 2} \left( 2 + \frac{1}{2} \right) = \frac{5}{8}
\end{align*}
\]

8. [4 marks]

If \( y^6 = 32x + y \ln x \) then when \( x = 1 \) and \( y = 2 \), \( \frac{dy}{dx} = \)

A. \(\frac{17}{40}\)
B. \(\frac{17}{10}\)
C. \(\frac{27}{40}\)
D. \(\frac{17}{16}\)
E. \(\frac{27}{26}\)

\[
\begin{align*}
5y^4 \frac{dy}{dx} &= 32 + \frac{y}{x} \\
\frac{dy}{dx} &= \frac{34}{80} = \frac{17}{40}
\end{align*}
\]
9. (4 marks)

If \( f(8) = 2 \) and \( f'(8) = 3 \), then
\[
\lim_{h \to 0} \frac{\ln(f(8+h)) - \ln(f(8))}{h} = \frac{\ln(f(8+h)) - \ln(f(8))}{h} \]

A. \( \ln 3 \)
B. 6
C. \( \frac{3}{2} \)
D. \( \frac{\ln 3}{2} \)
E. 3

\[
\frac{d}{dx} \ln(f(x)) \quad \text{at} \quad x = 8.
\]

\[
= \frac{f'(x)}{f(x)} \quad \text{at} \quad x = 8
\]

\[
= \frac{f'(8)}{f(8)} = \frac{3}{2}
\]


10. (4 marks)

If a revenue function is given, in dollars, by
\[
r(q) = (100 - 2 \ln q) q
\]
and a production function is given by
\[
q = 10(\sqrt{m} - 5)
\]

where \( m \) is the number of employees and \( q \) is the quantity produced, then, when \( m = 100 \), the marginal revenue product is closest to

A. \$230,440
B. \$4,609
C. \$115,220
D. \$2,304
E. \$45

\[
\frac{dr}{dq} \frac{dq}{dm} = (98 - 2 \ln q) \left( \frac{5}{\sqrt{m}} \right)
\]

When \( m = 100 \), \( q = 10 \cdot 5 = 50 \),

\[
\frac{dr}{dm} \bigg|_{m=100} = (98 - 2 \ln 50) \frac{1}{2}
\]

\( \approx 45.09 \)

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PART B. Written-Answer Questions

1. [16 marks]

(a) [7 marks]

Evaluate \( \lim_{t \to 2} \frac{|t - 2|}{|t| - 2} \) or show that it doesn’t exist.

**Hint:** You may want to consider each of the one sided limits first.

When \( t \to 2^+ \), \( t > 0 \); so \( |t| = t \)

\[
\lim_{t \to 2^+} \frac{|t - 2|}{|t| - 2} = \lim_{t \to 2^+} \frac{t - 2}{t - 2} = 1
\]

\[
\lim_{t \to 2^-} \frac{|t - 2|}{|t| - 2} = \lim_{t \to 2^-} \frac{-(t - 2)}{t - 2} = -1
\]

Since the limit from the right and the limit from the left are not equal, the limit itself does not exist.

Saying that \( \frac{0}{0} \) is undefined and "hence" the limit does not exist is worth NOTHING!
1. (b) [9 marks]

For what values of \( b \) and \( c \) is the function

\[
f(x) = \begin{cases} 
\frac{x^4 + bx^2 - 10x^2}{(x+c)(x-2)} & x > 2 \\
\frac{4}{x} & x \leq 2.
\end{cases}
\]

continuous for all values of \( x \)? Make sure to explain your answer properly.

For \( x > 2 \),

\[
f(x) = \frac{x^2(x^2 + bx - 10)}{(x+c)(x-2)}
\]

For \( f \) to even have a limit from the right at \( x = 2 \), \( x - 2 \) must be a factor of \( x^2 + bx - 10 = (x-2)(x-r) \)

so \( r = 5 \) to match the -10.

\[(x-2)(x+5) = x^2 + 3x - 10 \quad \text{so} \quad b = 3\]

\[
f(x) = \begin{cases} 
\frac{x^2(x+5)}{x+c} & x > 2 \\
\frac{4}{x} & x \leq 2
\end{cases}
\]

\[
\lim_{x \to 2^+} f(x) = \frac{28}{2+c}
\]

\[
\lim_{x \to 2^-} f(x) = 4 = f(2) \quad \text{by definition}
\]

For continuity everywhere, all that is now required is \( \frac{28}{2+c} = 4 \) so \( c = 5 \)

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2. [11 marks]

Three payments of $1,000 each are to be made into an account which earns 10% nominal annual interest compounded continuously. The second payment is to be made 14 months after the initial payment, and the third payment is to be made 34 weeks after the second payment.

Find the total value of all three payments calculated at the time the second payment is made.

[You may consider each month to be $\frac{1}{12}$ of a year and each week $\frac{1}{52}$ of a year.]

\[
\sqrt{1000e^{10\left(\frac{14}{12}\right)} + 1000 + 1000e^{-10\left(\frac{34}{52}\right)}}
\]

\[= \$3040.45\]
3. [18 marks]

Given the input-output matrix for the three industries

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<th>Industry C</th>
<th>Final Demand</th>
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<tr>
<td>A</td>
<td>40</td>
<td>0</td>
<td>25</td>
</tr>
<tr>
<td>B</td>
<td>80</td>
<td>60</td>
<td>25</td>
</tr>
<tr>
<td>C</td>
<td>0</td>
<td>120</td>
<td>75</td>
</tr>
<tr>
<td>Other Production Factors</td>
<td>80</td>
<td>120</td>
<td>125</td>
</tr>
</tbody>
</table>

[6] (a) Find the coefficient matrix (technology matrix).

[9] (b) Find the new output for each of the industries if final demand changes to 100 for each of the industries.

[8] (c) How much, in total, of the “Other Production Factors” is used to meet the new final demand?

\[ A \text{ has inputs from industry A to itself and others} \]
= total output of industry A = 40 + 25 + 135 = 200

Similarly, \[ B = 80 + 60 + 25 + 135 = 300 \]
\[ C = 120 + 75 + 55 = 250 \]

Col A has inputs from all industries to industry A. Dividing by total output of industry A gives input from each industry per unit out of industry A.

Similarly Col B and Col C for industries B and C.

\[ A = \begin{pmatrix}
40 & 0 & 25 \\
80 & 60 & 25 \\
0 & 120 & 75
\end{pmatrix} \]
\[ = \begin{pmatrix}
\frac{1}{5} & 0 & \frac{1}{10} \\
\frac{2}{5} & \frac{1}{5} & \frac{1}{10} \\
0 & \frac{2}{5} & \frac{3}{10}
\end{pmatrix} \]

Note: For part (c) we will also need to know
\[ \frac{80}{200} = \frac{2}{5} \text{ unit of "other" per unit of A} \]
\[ \frac{120}{300} = \frac{2}{5} \text{ unit of "other" per unit of B} \]
\[ \frac{125}{250} = \frac{1}{2} \text{ unit of "other" per unit of C} \]

Question 3 continues on Page 11
3b) \( I - A = \begin{pmatrix} \frac{4}{5} & 0 & -\frac{1}{10} \\ -\frac{3}{5} & \frac{4}{5} & -\frac{1}{10} \\ 0 & -\frac{3}{5} & \frac{7}{10} \end{pmatrix} \), solve \((I - A)X = \begin{pmatrix} 100 \\ 100 \end{pmatrix}\) where \(X = \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix}\)

Multiplying by 10,

\[
\begin{pmatrix} 8 & 0 & -1 \\ -4 & 8 & -1 \\ 0 & -4 & 7 \end{pmatrix} \begin{pmatrix} R_1 \\ R_2 \\ R_3 \end{pmatrix} \begin{pmatrix} 1000 \\ 1000 \\ 1000 \end{pmatrix}
\]

\[
R_1 \leftrightarrow R_2 \quad R_2 \rightarrow R_2 + R_1 \quad R_1 \rightarrow -\frac{1}{4} R_1
\]

\[
\begin{pmatrix} 1 & 2 & \frac{1}{4} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 25 \end{pmatrix} \begin{pmatrix} -250 \\ -250 \\ 7000 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & \frac{1}{4} \\ 0 & 1 & -\frac{3}{4} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} -250 \\ -250 \\ 280 \end{pmatrix}
\]

\[X_C = 280\]
\[X_B = -250 + \frac{3}{4} X_C = -250 + 490 = 240\]
\[X_A = -250 - \frac{1}{4} X_C + 2X_B = -250 - 70 + 480 = 160\]

\[X = \begin{pmatrix} x_A \\ x_B \\ x_C \end{pmatrix} = \begin{pmatrix} 160 \\ 240 \\ 280 \end{pmatrix}\]

3c) From \(\text{Note at the end of (a)}\),

other production factors = \(\frac{2}{5} \times 160 + \frac{2}{5} \times 240 + \frac{1}{2} \times 280 = 300\)
4. [15 marks]

The Powdered Rum Company is a monopoly which sells its product to navies the world over. When price \( p \) is given in dollars per gram and demand \( q \) is given in grams per second the demand for its product satisfies

\[ 2p^2 + 2pq + q^2 = 1000. \]

Note that \( q = 20 \) when \( p = 10 \).

[7] (a) Find \( \frac{dq}{dp} \) when \( p = 10 \).

\[
4p + 2q + 2p \frac{dq}{dp} + 2q \frac{dq}{dp} = 0 \Rightarrow 40 + 40 + 20 \frac{dq}{dp} + 40 \frac{dq}{dp} = 0
\]

or

\[
\frac{dq}{dp} = -\frac{2p+q}{p+q} = -\frac{40}{10} = -\frac{4}{3}
\]

[8] (b) Find marginal revenue with respect to price when \( p = 10 \).

That is, find \( \frac{dr}{dp} \) when \( p = 10 \), where \( r \) denotes revenue in dollars per second.

\[
r = pq
\]

\[
\frac{dr}{dp} = q + p \frac{dq}{dp}
\]

\[
= 20 + 10 \left(-\frac{4}{3}\right) \quad \text{when} \quad p = 10, \quad q = 20.
\]

\[
\frac{dr}{dp} = \frac{20}{3} \approx 6.67
\]

[9] (c) Find the approximate change in revenue which would result if the Company were to raise its price from 10 to 12 dollars per gram.

\[
\Delta r \approx \frac{dr}{dp} \Delta p = \frac{20}{3} \times 2 = \frac{40}{3} \approx $13.33
\]