1. \[
\lim_{h \to 0} \frac{(x + h)^n - x^n}{h} = \lim_{h \to 0} \frac{(x + h - x)((x + h)^{n-1} + (x + h)^{n-2}x + \ldots + x^{n-1})}{h} \\
= \lim_{h \to 0} ((x + h)^{n-1} + (x + h)^{n-2}x + \ldots + x^{n-1}) \\
= x^{n-1} + x^{n-1} + \ldots + x^{n-1} \quad \text{(n times)} \\
= nx^{n-1}
\]

2. (a) Let \(a = 0\), \(f(x) = \frac{1}{x}\), and \(g(x) = -\frac{1}{x}\).

(b) Let \(a = 0\), \(f(x) = \text{sgn}(x)\), and \(g(x) = \text{sgn}(x)\), where
\[
\text{sgn}(x) := \begin{cases} 
1 & x > 0 \\
0 & x = 0 \\
-1 & x < 0
\end{cases}
\]

3. (a) If \(c = 0\) then \(\lim_{x \to 0} \frac{\sqrt[3]{1 + cx} - 1}{x} = \lim_{x \to 0} \frac{0}{x} = 0 = \frac{c}{3}\). Otherwise,
\[
\lim_{x \to 0} \frac{\sqrt[3]{1 + cx} - 1}{x} = \lim_{x \to 0} \frac{c((\sqrt[3]{1 + cx} - 1))}{(1 + cx) - 1} \\
= \lim_{x \to 0} \frac{c((\sqrt[3]{1 + cx} - 1))}{(\sqrt[3]{(1 + cx)^2} + \sqrt[3]{1 + cx} + 1)} \\
= \lim_{x \to 0} \frac{c}{(\sqrt[3]{(1 + cx)^2} + \sqrt[3]{1 + cx} + 1)} = \frac{c}{3}
\]

(b)
\[
\lim_{x \to 1} \frac{\sqrt{x} - 1}{\sqrt{x} - 1} = \lim_{x \to 1} \frac{(\sqrt{x} - 1)(\sqrt{x} + 1)(\sqrt{x^2} + \sqrt[3]{x} + 1)}{(\sqrt{x} - 1)(\sqrt{x} + 1)(\sqrt{x^2} + \sqrt{x} + 1)} \\
= \lim_{x \to 1} \frac{(x - 1)(\sqrt{x} + 1)}{(x - 1)(\sqrt{x^2} + \sqrt[3]{x} + 1)} = \frac{2}{3}
\]

4. (a) Not possible to find with the given information.

(b) \(L\)
(c) Not possible to find with the given information.

(d) Not possible to find, $-L$, not possible to find.

5. (a)

i) When $x > 1$, $|x-1| = x-1$, and so $\lim_{x \to 1^+} F(x) = \lim_{x \to 1^+} \frac{(x-1)(x+1)}{x-1} = \lim_{x \to 1^+} x + 1 = 2$

ii) When $x < 1$, $|x-1| = -(x-1)$, and so $\lim_{x \to 1^-} F(x) = \lim_{x \to 1^-} \frac{(x-1)(x+1)}{-(x-1)} = \lim_{x \to 1^-} -(x+1) = -2$

(b) No. $\lim_{x \to 1^+} F(x) = 2 \neq -2 = \lim_{x \to 1^-} F(x)$, so the left-hand limit is not the same as the right-hand limit and thus $\lim_{x \to 1} F(x)$ does not exist.

(c) (i) When $x > 1$, $F(x) = x + 1$ (from (a) i)), and (ii) When $x < 1$, $F(x) = -(x + 1)$ (from (a) ii)), (iii) When $x = 1$, $F(x)$ does not exist, so we have:

6. (a) The slope of the line is $\frac{\Delta y}{\Delta x} = \frac{\ln(1+\frac{r}{n}) - \ln(1)}{(1+\frac{r}{n})-1} = \frac{\ln(1+\frac{r}{n})}{\frac{r}{n}} = \frac{n}{r} \ln(1 + \frac{r}{n})$.

(b) Writing $h = \frac{r}{n}$, we have $h \to 0$ as $n \to \infty$ so

$$\lim_{n \to \infty} \frac{n}{r} \ln(1 + \frac{r}{n}) = \lim_{h \to 0} \frac{\ln(1 + h)}{h}$$

$$= \lim_{h \to 0} \frac{\ln(1 + h) - \ln(1)}{h}$$

$$= g'(1) \quad (\text{where } g = \ln x)$$
\[
\lim_{n \to \infty} (1 + \frac{r}{n})^n = \lim_{n \to \infty} e^{n \ln(1 + \frac{r}{n})} = e^{r'g'(1)} \text{ since the function } e^x \text{ is continuous and we know } \lim_{n \to \infty} n \ln(1 + \frac{r}{n}) = r'g'(1) = e^r
\]

7. (a) For a principal amount of money, \( P \), \( P(1 + \frac{r}{n})^n \) is the sum of money resulting from compound interest, compounded \( n \) times at the interest rate \( r \), so we should have \( P(1 + \frac{r}{n})^n < P(1 + \frac{r}{n+1})^{n+1} \) which implies \((1 + \frac{r}{n})^n < (1 + \frac{r}{n+1})^{n+1} \).

(b) In the graph of \( y = \ln x \) we can see that \( \frac{n+1}{r} \ln(1 + \frac{r}{n+1}) > \frac{n}{r} \ln(1 + \frac{r}{n}) \), the slopes of the lines \( A \) and \( B \) respectively.

8. (a) \( f(x) = \begin{cases} 0; & 0 \leq x < 1 \\ 1; & x = 1 \end{cases} \), so \( f(x) \) is discontinuous at \( x = 1 \).

(b) \( f(x) = \begin{cases} 0; & 0 \leq x < 1 \\ 1; & x > 1 \end{cases} \), \( f(x) \) is undefined at \( x = 1 \). Thus \( f(x) \) is discontinuous at \( x = 1 \).

9. For \( m < 0 \), the function is undefined at \( x = 0 \) and is therefore not continuous there. For \( m = 0 \), \( f(x) = 1 \) and is therefore continuous everywhere. For \( m > 0 \), \( \lim_{x \to 0} x^m = 0 = f(0) \), so \( f(x) \) is continuous at \( x = 0 \). Therefore, it is required that \( m \geq 0 \) for \( f(x) \) to be continuous at \( x = 0 \).
10. (a) \( f(x) = (x - a)^m f_1(x) \), where \( f_1(a) \neq 0 \) \( g(x) = (x - a)^n g_1(x) \), where \( g_1(a) \neq 0 \) and \( m \geq n \geq 1 \).

(b) \( h(x) = \frac{(x-a)^{m-n} f_1(x)}{g_1(x)} \)

11. (a) When \( x^2 - 1 = 0 \), then \( f(x) \) is not defined. So we need only consider

i) when \( x^2 - 1 > 0 \). Here \( f(x) = \frac{x^2 - 1}{x^2 - 1} = 1 \) is continuous everywhere where \( x^2 - 1 > 0 \).

ii) when \( x^2 - 1 < 0 \). Similarly, \( f(x) = \frac{-(x^2 - 1)}{x^2 - 1} = -1 \) is continuous everywhere where \( x^2 - 1 < 0 \).

(b) \( f(x) \) is not defined when \( x^2 - 1 = 0 \), i.e., when \( x \pm 1 \)

i) If \( x = 1 \), then since

\[
A) \quad \lim_{x \to 1^-} \frac{|x^2 - 1|}{x^2 - 1} = \lim_{x \to 1^-} -\frac{x^2 - 1}{x^2 - 1} = -1, \quad \text{and}
\]

\[
B) \quad \lim_{x \to 1^+} \frac{|x^2 - 1|}{x^2 - 1} = \lim_{x \to 1^+} \frac{x^2 - 1}{x^2 - 1} = 1, \quad \text{then}
\]

no matter what our choice for \( f(1) \), \( f \) cannot be continuous at \( x = 1 \) since the left-hand and right-hand limits differ.

ii) Similarly, if \( x = -1 \), then

\[
A) \quad \lim_{x \to -1^-} \frac{|x^2 - 1|}{x^2 - 1} = \frac{x^2 - 1}{x^2 - 1} = 1, \quad \text{and}
\]

\[
B) \quad \lim_{x \to -1^+} \frac{|x^2 - 1|}{x^2 - 1} = -\frac{x^2 - 1}{x^2 - 1} = -1,
\]

hence, no value of \( f \) can be found at \( -1 \) to make \( f \) continuous at \( -1 \).