1. INTRODUCTION

The purpose of this handbook is to provide information about the graduate program of the Department of Mathematics, University of Toronto. It includes detailed information about the department, its faculty members and students, a listing of courses offered in 2005-2006, a summary of research activities, admissions requirements, application procedures, fees and financial assistance, and information about similar matters of concern to graduate students and prospective graduate students in mathematics.

This handbook is intended to complement the calendar of the university’s School of Graduate Studies, where full details on fees and general graduate studies regulations may be found.

For further information, or for application forms, please contact:

Ms. Ida Bulat
The Graduate Office
Department of Mathematics
University of Toronto
40 St. George Street, Room 6291
Toronto, Ontario, Canada M5S 3G3
Telephone: (416) 978-7894
Fax: (416) 978-4107
Email: grad-info@math.toronto.edu
Website: http://www.math.toronto.edu/graduate
2. DEPARTMENT OF MATHEMATICS

Mathematics has been taught at the University of Toronto since 1827. Since the first Canadian Ph.D. degree in mathematics was conferred to Samuel Beatty (under the supervision of John Charles Fields) in 1915, more than 350 Ph.D. degrees and 900 Master’s degrees have been awarded in this University. Many of our recent graduates are engaged in university teaching, and a significant number hold administrative positions in universities or in the professional communities. Others are pursuing careers in industry (technological or financial), or in government.

The Department of Mathematics, University of Toronto is a distinguished faculty of more than sixty mathematicians. We have a large selection of graduate courses and seminars, and a diverse student body of domestic and international students, yet classes are small and the ratio of graduate students to faculty is low. We are in a unique position to take maximum advantage of the presence of the Fields Institute, which features special programs in pure and applied mathematics. Currently the department has 103 graduate students, of whom 25 are enrolled in the Master’s program, and 78 in the Ph.D. program.

Opportunities for graduate study and research are available in most of the main fields of pure and applied mathematics. These fields include real and complex analysis, ordinary and partial differential equations, harmonic analysis, nonlinear analysis, several complex variables, functional analysis, operator theory, C*-algebras, ergodic theory, group theory, analytic and algebraic number theory, Lie groups and Lie algebras, automorphic forms, commutative algebra, algebraic geometry, singularity theory, differential geometry, symplectic geometry, classical synthetic geometry, algebraic topology, set theory, set theoretic topology, mathematical physics, fluid mechanics, probability (in cooperation with the Department of Statistics), combinatorics, optimization, control theory, dynamical systems, computer algebra, cryptography, and mathematical finance.

We offer a research-oriented Ph.D., and Master’s program. Very strong students may be admitted directly to the Ph.D. program with a Bachelor’s degree; otherwise, it is normal to do a 1-year Master’s degree first. (Provisional admission to the Ph.D. program may be granted at the time of admission to the Master’s program.) The Master’s program may be extended to 16 months or 24 months for students who do not have a complete undergraduate preparation, or for industrial students engaged in a project.

There is a separate Master’s of Mathematical Finance Program not directly under the Department’s jurisdiction, but with which some of our faculty members are associated.

During their studies here, graduate students are encouraged to participate in the life of the close community of U of T mathematics. Almost all of them do some work in connection with undergraduate teaching, either as tutorial leaders, markers, or, especially in later years of their program, instructors. There is a Mathematics Graduate Student Association which organizes social and academic events and makes students feel welcome.

GRADUATE FACULTY MEMBERS

AKCOGLU, M.A. (Professor Emeritus) Ph.D. 1963 (Brown)
  • Ergodic theory, functional analysis, harmonic analysis
ANGEL, O. (Assistant Professor) Ph.D. 2003 (Weizmann Institute of Science)
  • Probability theory
ARKHIPOV, S. (Assistant Professor) Ph.D. 1998 (Moscow State)
  • Geometric representation theory
ARTHUR, J. (University Professor) B.Sc. 1966 (Toronto), M.Sc. 1967 (Toronto), Ph.D. 1970 (Yale)
  • Representations of Lie groups, automorphic forms
BARBEAU, E. (Professor Emeritus) B.Sc. 1960 (Toronto), M.A. 1961 (Toronto), Ph.D. 1964 (Newcastle)
  • Functional analysis, optimization under constraint, history of analysis, number theory
  • Theory of quantum invariants of knots, links and three manifolds
BIERSTONE, E. (Professor) B.Sc. 1969 (Toronto), Ph.D. 1973 (Brandeis)
  • Singularity theory, analytic geometry, differential analysis
BINDER, I. (Assistant Professor) Ph.D. 1997 (Caltech)
  • Harmonic and complex analysis, conformal dynamics
BLAND, J. (Professor) Ph.D. 1982 (UCLA)
- Several complex variables, differential geometry
BLOMER, V. (Assistant Professor) Ph.D. 2002 (Stuttgart)
- Analytic number theory
BLOOM, T. (Professor) Ph.D. 1965 (Princeton)
- Several complex variables
BUCHWEITZ, R.-O. (Professor) Ph.D. (Dr.rer.nat.) 1976 (Hannover), Doctorat d’Etat 1981 (Paris VII)
- Commutative algebra, algebraic geometry, singularities
BURCHARD, A. (Associate Professor) Ph.D. (Georgia Tech) 1994
- Functional analysis
BUTSCHER, A. (Assistant Professor) 2000 (Stanford)
- Geometric analysis
CHO, C-H. (Assistant Professor) Ph.D. 2003 (University of Wisconsin-Madison)
- Differential geometry, Quantum theory
CHOI, M.-D. (Professor) M.Sc. 1970 (Toronto), Ph.D. 1973 (Toronto)
- Operator theory, operator algebras, matrix theory
COLLIANDER, James (Associate Professor) Ph.D. 1997 (Illinois, Urbana-Champaign)
- Partial differential equations, harmonic analysis
- Operators on Hilbert spaces, matrix theory and applications (including numerical analysis)
DEL JUNCO, A. (Professor) B.Sc. 1970 (Toronto), M.Sc. 1971 (Toronto), Ph.D. 1974 (Toronto)
- Ergodic theory, functional analysis
DERZKO, N. (Associate Professor) B.Sc. 1970 (Toronto), Ph.D. 1965 (Caltech)
- Functional analysis, structure of differential operators, optimization and control theory with applications to economics
ELLERS, E. (Professor Emeritus) Dr.rer.nat. 1959 (Hamburg)
- Classical groups
ELLIOIT, G. A. (Canada Research Chair and Professor) Ph.D. 1969 (Toronto)
- Operator algebras, K-theory, non-commutative geometry and topology
FORNI, G. (Professor) Ph.D. 1993 (Princeton)
- Dynamical systems and ergodic theory
FRIEDLANDER, J. (University Professor) B.Sc. 1965 (Toronto), Ph.D. 1972 (Penn State)
- Analytic number theory
- Spectral theory of Schroedinger operators and localization
GRAHAM, I. (Professor) B.Sc. 1970 (Toronto), Ph.D. 1973 (Princeton)
- Several complex variables, one complex variable
GREINER, P.C. (Professor Emeritus) Ph.D. 1964 (Yale)
- Partial differential equations
HALPERIN, S. (Professor Emeritus) B.Sc. 1965 (Toronto), M.Sc. 1966 (Toronto), Ph.D. 1970 (Cornell)
- Homotopy theory and loop space homology
HORI, K. (Assistant Professor) Ph.D. 1994 (Tokyo)
- Gauge field theory, string theory
IVRII, V. (Professor) Ph.D. 1973 (Novosibirsk)
- Partial differential equations
JEFFREY, L. (Professor) Ph.D. 1992 (Oxford)
- Symplectic geometry, geometric applications of quantum field theory
JERRARD, Robert (Professor) Ph.D. 1994 (Berkeley)
- Nonlinear partial differential equations, Ginzburg-Landau theory
JURDJEVIC, V. (Professor Emeritus) Ph.D. 1969 (Case Western)
- Systems of ordinary differential equations, control theory, global analysis
KAPOVitch, V. (Associate Professor) Ph.D. 1997 (University of Maryland)
- Global riemannian geometry
KARSHON, Y. (Professor) Ph.D. 1993 (Harvard)
- Equivariant symplectic geometry
KAVEH, K. (Assistant Professor) Ph.D. 2002 (University of Toronto)
- Algebraic group

KEYFITZ, B. (Adjunct Full Professor) Ph.D. 1970 (New York University)
- Nonlinear partial differential equations

KHANIN, K. (Professor) Ph.D. 1983 (Landau Institute, Moscow)
- Dynamical systems and statistical mechanics

Khesin, B. (Professor) Ph.D. 1989 (Moscow State)
- Poisson geometry, integrable systems, topological hydrodynamics

KHOVANSKII, A. (Professor) Ph.D. 1973, Doctorat d’Etat 1987 (Steklov Institute, Moscow)
- Algebra, geometry, theory of singularities

KIM, Henry (Professor) Ph.D. 1992 (Chicago)
- Automorphic L-functions, Langlands’ program

KUDLA, S. (Senior Canada Research Chair and Professor) Ph.D. 1971 (Harvard)
- Automorphic forms, Arithmetic geometry and Theta functions

LORIMER, J.W. (Professor) Ph.D. 1971 (McMaster)
- Rings and geometries, topological Klingenberg planes, topological chain rings

LYUBICH, M. (Canada Research Chair and Professor) Ph.D. 1984 (Tashkent State University)
- Dynamical systems, especially holomorphic and low dimensional dynamics

- Special functions, Jacobi matrices, orthogonal polynomials, difference equations, continued fractions and q-series

McCANN, R. (Professor) Ph.D. 1994 (Princeton)
- Mathematical physics, mathematical economics, inequalities, optimization, partial differential equations

McCooL, J. (Professor Emeritus) Ph.D. 1966 (Glasgow)
- Infinite group theory

MEINRENKEN, E. (Professor) Ph.D. 1994 (Universität Freiburg)
- Symplectic geometry

MENDELSON, E. (Professor) Ph.D. 1968 (McGill)
- Block designs, combinatorial structures

MIKHALKIN, G. (Professor) Ph.D. 1993 (Michigan State University)
- Geometry, topology and algebraic geometry

MILMAN, P. (Professor) Ph.D. 1975 (Tel Aviv)
- Singularity theory, analytic geometry, differential analysis

MURASUGI, K. (Professor Emeritus) D.Sc. 1960 (Tokyo)
- Knot theory

MURNAGHAN, F. (Professor) Ph.D. 1987 (Chicago)
- Harmonic analysis and representations of p-adic groups

- Number theory

NABUTOVSKY, A. (Associate Professor) Ph.D. 1992 (Weizmann Institute of Science)
- Geometry and logic

NACHMAN, A. (Professor) Ph.D. 1980 (Princeton)
- Inverse problems, partial differential equations, medical imaging

QUASTEL, J. (Professor) Ph.D. 1990 (Courant Institute)
- Probability, stochastic processes, partial differential equations

PONGE, R. (Assistant Professor) Ph.D. 2000 (University of Paris-Sud)
- Global analysis, analysis on manifolds, Several complex variables and analytic spaces

PUGH, C. (Distinguished Visiting Professor) Ph.D. (Johns Hopkins) 1965
- Dynamics and topology

PUGH, M. (Associate Professor) Ph.D. 1993 (Chicago)
- Scientific computing, nonlinear PDEs, fluid dynamics, computational neuroscience

RANGER, K.B. (Professor Emeritus) Ph.D. 1959 (London)
- Fluid mechanics, applied analysis with particular reference to boundary value problems, biofluid dynamics

REPKA, J. (Professor) B.Sc. 1971 (Toronto), Ph.D. 1975 (Yale)
- Group representations, automorphic forms
ROONEY, P.G. (Professor Emeritus) Ph.D. 1952 (Caltech)
- Integral operators, functional analysis

ROSENTHAL, P. (Professor) Ph.D. 1967 (Michigan)
- Operators on Hilbert spaces

ROTMAN, R. (Assistant Professor) Ph.D. 1998 (SUNY, Stony Brook)
- Riemannian geometry

SCHERK, J. (Associate Professor) D.Phil. 1978 (Oxford)
- Algebraic geometry

SECO, L. (Professor) Ph.D. 1989 (Princeton)
- Harmonic analysis, mathematical physics, mathematical finance

SELICK, P. (Professor) B.Sc. 1972 (Toronto), M.Sc. 1973 (Toronto), Ph.D. 1977 (Princeton)
- Algebraic topology

SEN, D.K. (Professor Emeritus) Dr.es.Sc. 1958 (Paris)
- Relativity and gravitation, mathematical physics

SHARPE, R. (Professor Emeritus) B.Sc. 1965 (Toronto), M.Sc. 1966 (Toronto), Ph.D. 1970 (Yale)
- Differential geometry, topology of manifolds

SHERK, F.A. (Professor Emeritus) Ph.D. 1957 (Toronto)
- Finite and discrete geometry

SHUB, M. (Professor) Ph.D. 1967 (Berkeley)
- Dynamical systems and complexity theory

SIGAL, I.M. (University Professor, Norman Stuart Robertson Chair in Applied Math) Ph.D. 1975 (Tel Aviv)
- Mathematical physics

- Fluid mechanics, particularly boundary layer theory

- Partial differential equations, nonlinear analysis, numerical computations in fluid dynamics

SZEGEDY, B. (Assistant Professor) Ph.D. 2002 (Eötvös University, Budapest)
- Group theory, Combinatorics, Computer science

TALL, F.D. (Professor) Ph.D. 1969 (Wisconsin)
- Set theory and its applications, set-theoretic topology

TANNY, S.M. (Associate Professor) Ph.D. 1973 (M.I.T.)
- Combinatorics, mathematical modeling in the social sciences

TODORCEVIC, S. (Canada Research Chair and Professor) Ph.D. 1979 (Belgrade)
- Set theory and combinatorics

VIRAG, B. (Junior Canada Research Chair and Assistant Professor) Ph.D. 2000 (Berkeley)
- Probability

WEISS, W. (Professor) M.Sc. 1972 (Toronto), Ph.D. 1975 (Toronto)
- Set theory, set-theoretic topology

YAMPOLSKY, M. (Associate Professor) Ph.D. 1997 (SUNY, Stony Brook)
- Holomorphic and low-dimensional dynamical systems

3. THE GRADUATE PROGRAM

The Department of Mathematics offers graduate programs leading to Master of Science (M.Sc.) and Doctor of Philosophy (Ph.D.) degrees in mathematics, in the fields of pure mathematics and applied mathematics.

The M.Sc. Program

The M.Sc. program may be done on either a full- or part-time basis. Full-time students normally complete the program in one full year of study; part-time students may take up to three years to complete the program. The degree requirements are as follows:

1a. Completion of 6 half-courses (or the equivalent combination of half- and full-year courses), as described in section 4 of this handbook. The normal course load for full-time graduate students is 3 courses in the fall term and 3 in the spring term. Students who intend to proceed in the Ph.D. program are required to take one full core course, and are strongly advised to take two of the core courses.
1b. Completion of the Supervised Research Project (MAT 4000Y). This project is intended to give the student the experience of independent study in some area of advanced mathematics, under the supervision of a faculty member. The supervisor and the student, with the approval of the graduate coordinator decide the topic and program of study. The project is normally undertaken during the summer session, after the other course requirements have been completed, and has a workload roughly equivalent to that of a full-year course.

2. M.Sc. Thesis Option (less common than option 1). Students who take this option will be required to take and pass two approved full-year courses and submit an acceptable thesis.

The Ph.D. Program

The Ph.D. program normally takes three or four years of full-time study beyond the Master’s level to complete. A Master’s degree is normally a prerequisite; however, exceptionally strong B.Sc. students may apply for direct admission to the Ph.D. program. Expected progress in the program is outlined in the following table:

<table>
<thead>
<tr>
<th>Year</th>
<th>Activities</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year 1</td>
<td>Completion of course work; Pass the comprehensive exams; Select a thesis advisor.</td>
</tr>
<tr>
<td>Year 2</td>
<td>Supervisory committee selected by graduate coordinator; First annual supervisory committee progress report due.</td>
</tr>
<tr>
<td>Year 3</td>
<td>Presentation of preliminary thesis results to supervisory committee.</td>
</tr>
<tr>
<td>Year 4</td>
<td>Thesis Content Seminar; Departmental PhD Thesis Examination; Final PhD Thesis Examination at the School of Graduate Studies.</td>
</tr>
</tbody>
</table>

1. **Coursework:** Completion of at least 6 half-courses (or the equivalent combination of half- and full-year courses), as described in section 4 of this handbook. Normally, 6 half-courses are taken in the first year of study (3 half-courses in the fall term and 3 in the spring term). It is strongly recommended that the student take some additional courses in other years.

2. **Comprehensive Examination:** The student is required to pass comprehensive examinations in basic mathematics before beginning an area of specialization. The examinations in the four general areas (real analysis, complex analysis, algebra, and topology) take place during a one-week period in early September. Exemptions from individual exams will be given if the student has obtained a grade of A- or better in the corresponding core course(s). Syllabi for the pure mathematics comprehensive exams appear in Appendix A. Copies of mock examination questions and/or past written examination papers are accessible to all candidates.

Students with interests in applied mathematics should refer to Appendix B for possible alternate comprehensive exams.

All exams are to be taken within 13 months of entering the Ph.D. program unless the Examination Committee grants permission in writing for a deferral.

3. **Supervisory Committee:** In accordance with School of Graduate Studies regulations, a supervisory committee will be established for each Ph.D. student who has chosen a research area and a supervisor. This committee consists of three faculty members including the supervisor. It is responsible for monitoring the student’s progress on an annual basis. By the end of the third year of Ph.D. studies, a student is required to present preliminary results of his/her research work to the supervisory committee. Information about general graduate supervision is available in the SGS document Graduate Supervision, Guidelines for Students, Faculty, and Administrators.

4. **Thesis:** The main requirement of the degree is an acceptable thesis. This will embody original research of a standard that warrants publication in the research literature. It must be written under the supervision of one or more members of the department. The student presents the thesis results in three stages.
(i) Thesis Content Seminar. This is an opportunity for the student to present his/her thesis results to department members. The presentation frequently takes place within one of the regular departmental research seminars.

(ii) Departmental Oral Examination. The student gives a 20-minute summary of the thesis and must defend it before a departmental examination committee. Copies of the thesis should be available two weeks before the departmental oral examination. The committee may approve the thesis without reservations, or approve the thesis on condition that revision be made, or require the student to take another departmental oral examination.

(iii) Final Oral Examination. Eight weeks after the successful completion of the departmental oral, the student proceeds to the final oral examination conducted by the School of Graduate Studies. The thesis is sent to an external reader who submits a report two weeks prior to the examination; this report is circulated to members of the examination committee and to the student. The examination committee consists of four to six faculty members; it is recommended that the external reader attend the examination. The student gives a 20-minute summary of the thesis which is followed by a question period.

5. Students are expected to become extensively involved in departmental life (seminars, colloquia and related activities).

Administration of the Graduate Program

A central administration authority called the School of Graduate Studies establishes the basic policies and procedures governing all graduate study at the University of Toronto. Detailed information about the School is obtained in its calendar, distributed to all graduate students during registration week, and in the admissions package that accompanies the application.

The Department of Mathematics has its own graduate administrative body—the graduate committee—composed of 6-8 faculty members appointed by the chair of the department, and five graduate students elected by the Mathematics Graduate Students Association. One of the faculty members is the graduate coordinator, who is responsible for the day-to-day operation of the program. The graduate committee meets frequently throughout the year to consider matters such as admissions, scholarships, course offerings, and departmental policies pertaining to graduate students. Student members are not permitted to attend meetings at which the agenda concerns confidential matters relating to other students. Information regarding appeals of academic decisions is given in the Grading Procedures section of the Calendar of the School of Graduate Studies. Students may also consult the Graduate Coordinator (or the student member of the departmental Graduate Appeals Committee) regarding information about such appeals.

General Outline of the 2006-2007 Academic Year

Registration August 14 – September 15, 2006
Fall Term Monday, September 11 – Friday, December 8, 2006
Spring Term Monday, January 8 – Thursday, April 13, 2007
Reading Week February 19 – 23, 2007

Official Holidays (University Closed):

Labour Day Monday, September 4, 2006
Thanksgiving Day Monday, October 9, 2006
Christmas/New Year Friday, December 22, 2006 – Wednesday, January 3, 2007
Good Friday Friday, April 6, 2007
Victoria Day Monday, May 21, 2007
Canada Day Monday, July 2, 2007
Civic Holiday Monday, August 6, 2007
4. GRADUATE COURSES

The following is a list and description of the courses offered to graduate students in the 2005-2006 academic year. Some of these courses are cross-listed with senior (400-level) undergraduate courses; that is, the courses are open to both graduate and senior undergraduate students. Others are intended for graduate students. Four of the courses are core courses. These are the basic beginning graduate courses. They are designed to help the student broaden and strengthen his/her general background in mathematics prior to specializing towards a thesis. A student with a strong background in the area of any of the core courses should not take that particular course. Some students will take all four core courses before the Ph.D. Comprehensive Examination; most will take some but not all of them. In addition, graduate students may take several intermediate (300-level) undergraduate courses (listed in the Faculty of Arts and Science Calendar) if their background is felt to be weak in some area; no graduate course credit is given for these courses.

There are three other means by which graduate students may obtain course credit, apart from completing the formal courses listed on the following pages. In each of these cases, prior approval of the graduate coordinator is required.

1. Students may take a suitable graduate course offered by another department. Normally at least two-thirds of the course requirements for each degree should be in the Mathematics Department.
2. It is sometimes possible to obtain course credit for appropriately extensive participation in a research seminar (see Research Activities section).
3. It is also possible to obtain a course credit by working on an individual reading course under the supervision of one of the faculty members, provided the material covered is not available in one of the formal courses or research seminars. (Note: this is distinct from the MAT 4000Y Supervised Research Project required of all M.Sc. students.)

Most courses meet for three hours each week, either in three one-hour sessions or two longer sessions. For some courses, particularly those cross-listed with undergraduate courses, the times and locations of classes will be set in advance of the start of term. For other courses, the times and locations of classes will be established at organizational meetings during the first week of term, so that a time convenient for all participants may be arranged. During registration week, students should consult the math department website or the graduate bulletin board outside the mathematics office for class and organization meeting times and locations.

CORE COURSES

MAT 1000YY (MAT 457Y1Y)
REAL ANALYSIS
G. Forni

1. Lebesgue integration, measure theory, convergence theorems, the Riesz representation theorem, Fubini’s theorem, complex measures.
2. $L^p$-spaces, density of continuous functions, Hilbert space, weak and strong topologies, integral operators.
3. Inequalities.
4. Bounded linear operators and functionals. Hahn-Banach theorem, open-mapping theorem, closed graph theorem, uniform boundedness principle.
5. Schwartz space, introduction to distributions, Fourier transforms on the circle and the line (Schwartz space and $L_2$).

Textbooks:

References:

**MAT 1001HS (MAT 454H1S)**

**COMPLEX ANALYSIS**

**E. Bierstone**

3. Conformal mapping, Riemann mapping theorem.

**References:**

H. Cartan: *Elementary theory of analytic functions of one or several complex variables*, Dover.

**MAT 1060HF**

**PARTIAL DIFFERENTIAL EQUATIONS I**

**R. McCann**

This course is a basic introduction to partial differential equations. It is meant to be accessible to beginners with little or no prior knowledge of the field. It is also meant to introduce beautiful ideas and techniques which are part of most analysts' basic bag of tools.

Some topics to be covered:

2. Sobolev spaces on \( \mathbb{R}^n \). Sobolev spaces on bounded domains. Weak solutions.

**Textbook:**

Lawrence Evans: *Partial Differential Equations*

**MAT 1061HS**

**PARTIAL DIFFERENTIAL EQUATIONS II**

**J. Colliander**

This course will consider a range of mostly nonlinear partial differential equations, including elliptic and parabolic PDE, as well as hyperbolic and other nonlinear wave equations. In order to study these equations, we will develop a variety of methods, including variational techniques, several fixed point theorems, and nonlinear semigroup theory. A recurring theme will be the relationship between variational questions, such as critical Sobolev exponents, and issues related to nonlinear evolution equations, such as finite-time blowup of solutions and/or long-time asymptotics.

The prerequisites for the course include familiarity with Sobolev and other function spaces, and in particular with fundamental embedding and compactness theorems.
MAT 1100YY
ALGEBRA
V. Blomer

1. Linear Algebra. Students will be expected to have a good grounding in linear algebra, vector spaces, dual spaces, direct sum, linear transformations and matrices, determinants, eigenvectors, minimal polynomials, Jordan canonical form, Cayley-Hamilton theorem, symmetric, alternating and Hermitian forms, polar decomposition.
2. Group Theory. Isomorphism theorems, group actions, Jordan-Hölder theorem, Sylow theorems, direct and semidirect products, finitely generated abelian groups, simple groups, symmetric groups, linear groups, nilpotent and solvable groups, generators and relations.
4. Modules. Modules and algebras over a ring, tensor products, modules over a principal ideal domain, applications to linear algebra, structure of semisimple algebras, application to representation theory of finite groups.
5. Fields. Algebraic and transcendental extensions, normal and separable extensions, fundamental theorem of Galois theory, solution of equations by radicals.

Textbooks:
Dummit and Foote: *Abstract Algebra*, 2nd Edition
Cohn: *Basic Algebra*
Other References:
Jacobson: *Basic Algebra, Volumes I and II.*
Lang: *Algebra.*
M. Artin: *Algebra.*

MAT 1300YY
TOPOLOGY
L. Jeffrey
P. Selick

A selection of topics from:
1. Set theory: Zorn's Lemma, axiom of choice, well-ordered sets, cardinals, ordinals
2. Point set topology: Metric spaces, topological spaces, compactness, separation properties, connectedness, paracompactness, CW complexes
3. Homotopy theory: homotopy, fundamental group, covering spaces, Van Kampen's theorem
4. Homological algebra: categories and functors, chain complexes, homology, exact sequences, Snake Lemma, Mayer-Vietoris
5. Homology theory: Eilenberg-Steenrod homology axioms; singular homology theory; cellular homology, cohomology, cup and cap products, applications of homology (Brouwer fixed-point theorem, vector fields on spheres, Jordan Curve Theorem), other homology theories (simplicial and Cech homology)
6. Manifolds: classification of surfaces, de Rham cohomology, orientation, Poincare Duality

Textbooks:
1. Munkres: "Topology"
2. Hatcher: "Algebraic Topology"
Other references:
1. Dugundji: "Topology"
2. Massey: "A basic course in algebraic topology"
4. Bott-Tu: "Differential Forms in Algebraic Topology"
5. Selick: "Introduction to homotopy theory"

2006-2007 CROSS-LISTED COURSES
MAT 1194HF (MAT 449H1F)
REAL ALGEBRAIC GEOMETRY
G. Mikhalkin

In the title of this course the term "real" stands not only as a reference to the field of real numbers but also as a reference to the real (geometric) world. In this course in addition to real varieties we shall also treat complex varieties (and sometimes even varieties defined over some function fields), but we shall primarily focus on geometric aspects of algebraic geometry. We'll pay special attention to topology of real and complex algebraic varieties, in particular to the topics related to Hilbert's 16th problem and to vanishing cycles of complex singularities. We introduce the so-called "patchworking" technique for constructing real algebraic from simpler pieces. The patchworking serves as a link from this course to the second half-course "Tropical Geometry".

Prerequisite:
Acquaintance with some basic Algebraic Geometry and Geometric Topology. New Ph.D. students are welcome, especially if they like Geometry. Please contact the instructor.

MAT 1302HS (APM 461H1S/CSC 2413HS)
COMBINATORIAL METHODS
S. Tanny

We will cover a selection of topics in enumerative combinatorics, such as more advanced methods in recursions, an analysis of some unusual self-referencing recursions, binomial coefficients and their identities, some special combinatorial numbers and their identities (Fibonacci, Stirling, Eulerian), and a general approach to the theory of generating functions.

Prerequisite:
Linear algebra.
Recommended preparation:
an introductory combinatorics course, such as MAT 344H.

MAT 1340HF (MAT 425H1F)
DIFFERENTIAL TOPOLOGY
A. Nabutovsky


Textbook:
Victor Guillemin and Alan Pollack, Differential Topology

Prerequisites: MAT 257Y (second year Analysis course), MAT 240H (second year Algebra course) and MAT 327H (introductory Topology course).

MAT 1342HS(MAT 464H1S)
DIFFERENTIAL GEOMETRY
J. Bland

**References:**
Manfredo Perdigao de Carmo: Riemannian Geometry.

**Prerequisite:** Introductory differential geometry course such as MAT 363H.

**MAT 1404HF (MAT 409H1F)**  
**SET THEORY**  
*W. Weiss*

We will introduce the basic principles of axiomatic set theory, leading to the undecidability of the continuum hypothesis. We will also explore those aspects of infinitary combinatorics most useful in applications to other branches of mathematics.

**Prerequisite:** an introductory real analysis course such as MAT 357H  
**Textbooks:**  
W. Just and M. Weese: *Discovering Modern Set Theory, I and II*, AMS.  

**MAT 1508HS (APM 446H1S)**  
**APPLIED NONLINEAR EQUATIONS**  
*A. Burchard*


**Prerequisite:**  
APM346H1/APM351Y1

**MAT 1700HS (APM 426H1S)**  
**GENERAL RELATIVITY**  
*P. Blue*


**Prerequisites:**  
Thorough knowledge of linear algebra and multivariable calculus. Some familiarity with partial differential equations, topology, differential geometry, and/or physics will prove helpful.

**Reference:**  
R. Wald, *General Relativity*, University of Chicago Press

**MAT 1723HF (APM 421H1F)**  
**MATHEMATICAL CONCEPTS OF QUANTUM MECHANICS AND QUANTUM INFORMATION**
I. M. Sigal

The goal of this course is to explain key concepts of Quantum Mechanics and to arrive quickly to some topics which are at the forefront of active research, such as Bose-Einstein condensation, control of chemical reactions and quantum information. We will try to be as self-contained as possible and rigorous whenever the rigour is instructive. Whenever the rigorous treatment is prohibitively time-consuming we give an idea of the proof, if such exists, and/or explain the mathematics involved without providing all the details.

Prerequisites for this course: some familiarity with elementary ordinary and partial differential equations and elementary theory of functions and operators.

Syllabus:
- Schroedinger equation
- Quantum observables
- Spectrum and evolution
- Density matrix and open systems
- Bose-Einstein condensation
- Quasiclassical asymptotics
- Path integral
- Approximate methods
- Hartree-Fock theory
- Control of chemical reactions
- Quantum entropy
- Quantum channels and information processing
- Basic notions of quantum field theory

References:
S. Gustafson and I. M. Sigal, Mathematical Concepts of Quantum Mechanics, Springer

MAT 1856HS (APM 466H1S)
MATHEMATICAL THEORY OF FINANCE
Instructor: TBA

Introduction to the basic mathematical techniques in pricing theory and risk management: Stochastic calculus, single-period finance, financial derivatives (tree-approximation and Black-Scholes model for equity derivatives, American derivatives, numerical methods, lattice models for interest-rate derivatives), value at risk, credit risk, portfolio theory.

2006-2007 TOPICS COURSES

JEB 1433HS
MEDICAL IMAGING
A. Nachman

This course will focus on Mathematical Methods in Medical Imaging. It will be accessible to beginning graduate students; topics and individual projects will be tailored to the background and interests of the class. Topics will be chosen from among the following:
- The multidimensional Fourier transform. Reconstruction from Fourier transform samples, Nyquist's theorem, Poisson summation.
• The Radon transform. Reconstruction from Radon transform samples.
• How does MRI work? The Bloch Equation in Magnetic Resonance Imaging; connection to the analytic development of the Inverse Scattering Transform (also used in the exact solution of integrable nonlinear partial differential equations).
• Current Density Impedance Imaging.
• (time permitting) Partial differential equations techniques for Ultrasound Imaging and Terahertz Tomography.

Texts:

Other reference:

Marking scheme: 75% project, 25% homework.

MAT 1035HF
INTRODUCTION TO C*-ALGEBRAS
M.-D. Choi

This course is concerned with the basic aspects of C*-algebras. During the first half of the semester, the lectures will be devoted to a systematic investigation of the finite-dimensional case—namely, the theory of $n \times n$ complex matrices. Through many simple concrete examples we may describe various phenomena arising from the interplay of normed structure, order structure and algebraic structure. Later in the semester, we will continue to explore more striking facts pertinent to the infinite-dimensional case. Based on the background of students, various topics of related interest will be pursued.

MAT 1045HS
INTRODUCTION TO (SMOOTH) ERGODIC THEORY
G. Forni

The course is an introduction to some of the basic notions and methods of the theory of dynamical systems. It covers notions of topological dynamics and ergodic theory such as minimality, topological transitivity, ergodicity, unique ergodicity, weak mixing and mixing. These notions will be explained by examining simple concrete examples of dynamical systems such as translations and automorphisms of tori, expanding maps of the interval, topological Markov chains, etc. Fundamental theorems of ergodic theory such as the Poincare recurrence theorem, the Von Neumann mean ergodic theorem and the Birkhoff ergodic theorem will be presented. Time permitting we will outline either entropy theory (topological entropy and Kolmogorov-Sinai entropy) and/or the subadditive ergodic theorem, the theory of Lyapunov exponents and the Oseledets theorem (with examples).

Prerequisites:
Knowledge of real analysis, basic topology and measure theory. Some knowledge of functional analysis would be useful.
HARMONIC ANALYSIS, DIOPHANTINE EQUATIONS AND INEQUALITIES
M. Goldstein


References:
Natanson, M., Additive Number Theory,
Montgomery, H., Ten Lectures on the Interface between Analytic Number Theory and Harmonic Analysis
Schmidt, W., Diophantine Approximations

Prerequisites:
Introductory courses in complex analysis (e.g. MAT 354H) and real analysis (e.g. MAT 357H).

MAT 1062HS
NUMERICAL METHODS FOR PARTIAL DIFFERENTIAL EQUATIONS
C. Sulem

GOAL: The goal is to introduce some basic numerical techniques for solving ordinary and partial differential equations. The course is intended to students in Applied Mathematics and Engineering who wish to learn numerical methods useful for their research.

1. Introduction: Interpolation and Approximation techniques. Numerical differentiation and integration

No prior knowledge is required, although a basic undergraduate course in ODEs or/and PDEs would be useful.

MAT 1103HF
GALOIS THEORY AND RIEMANN SURFACES
A. Khovanskii

Galois Theory belongs to algebra. It is very understandable and has a lot of applications. For example it explains why algebraic equations are usually not solvable by means of radicals.

About 30 years ago I constructed a topological version of Galois Theory for functions in one complex variable. According to it, there are topological restrictions on the way the Riemann surface of a function representable by radicals covers the complex plane. If the function does not satisfy these restrictions, then it is not representable by radicals. Beside its geometric clarity the topological results on nonsolvability are stronger than the algebraic results. They have a lot of generalizations.

In the course I plan to present Galois Theory in details, to discuss topological results on nonsolvability by radical and their generalizations, to give an introduction to a multidimensional version of the theory.

Prerequisite:
Some basic knowledge in complex analysis and in elementary algebra.

MAT 1104HS
K-THEORY AND C*-ALGEBRAS
G. Elliott
The theory of algebras of operators in Hilbert space has a distinctly topological character, and in particular the K-theory of Atiyah and Hirzebruch is useful. Indeed, this theory finds its original roots (even before the work of Grothendieck in algebraic geometry) in the work of Murray and von Neumann that began the subject of operator algebras. What might be called the second-generation application of this tool to operator algebras, incorporating Bott periodicity (as formulated in terms of K-theory by Atiyah and Hirzebruch), has made it possible to study both the intrinsic structure and the classification of C*-algebras, and also to obtain generalizations of the index theorem of Atiyah and Singer.

**Prerequisites:**
The spectral theorem

**References:**
M. Rordam, *K-Theory for C*-Algebras*
H. Lin, *An Introduction to the Classification of Amenable C*-Algebras*
A. Connes, *Noncommutative Geometry*

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**MAT 1126HF**
**INTRODUCTION TO NONHOLONOMIC MECHANICS AND GEOMETRY**

**B. Khesin**

Nonholonomic mechanics describes the geometry of systems subordinated to nonholonomic constraints, i.e. systems whose restrictions on velocities do not arise from the constraints on the configuration space. The best known examples of such systems are a sliding skate and a rolling ball, as well as their numerous generalizations.

We start with an introduction to the Euler-Lagrange equation and the Lagrange-d'Alambert principle. After defining Hamiltonian systems and their integrability we go through

1. main nonholonomic examples (integrable and not);
2. reduction of symmetries in nonholonomic mechanics;
3. main concepts of the nonholonomic (i.e. sub-Riemannian) differential geometry: geometry of distributions, their curvatures, Carnot-Caratheodory metrics and their geodesics, singular geodesics;
4. relation to geometric mechanics and control theory.

**References:**

**Prerequisites:**
Some knowledge of Hamiltonian systems/symplectic geometry or MAT 1051HF (MAT 468H1F) is helpful.

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**MAT 1155HF**
**INTRODUCTION TO COMMUTATIVE ALGEBRA AND ALGEBRAIC GEOMETRY**

**S. Kudla**

This course will provide a basic introduction to algebraic geometry and the underlying commutative algebra. Topics will include:

- Affine varieties: Nullstellensatz, Zariski topology, ideal theory in Noetherian rings, Krull dimension, examples
- projective varieties: graded rings and ideals, P^n, examples, cones, quadrics, Grassmannians
• morphisms: regular functions, localization, local rings, rational functions, morphisms, products, examples
• rational maps: dominant and birational morphisms, birational equivalence, blowing up, examples
• nonsingular varieties: singular and nonsingular points, regular local rings, completions, analytic isomorphism, curve singularities,
• nonsingular curves: function fields, valuations, discrete valuation rings, fundamental theorem on nonsingular curves and function fields, examples.
• intersection theory: dimensions, Hilbert polynomials, intersection multiplicities, Bezout's theorem

Basic reference:
R. Hartshorne, Algebraic Geometry, Chapter I.

Additional references:
W. Fulton, Algebraic curves: an introduction to algebraic geometry.
D. Mumford, The red book of varieties and schemes. LN 1358
_________, Algebraic geometry I: complex projective varieties.

Prerequisite:
One year of algebra at a graduate level, e.g. MAT 1100Y.

MAT 1191HS
TROPICAL GEOMETRY
G. Mikhalkin

This course is intended to introduce students to a recent technique in Algebraic Geometry based on application of the moment map and toric degenerations. One of the simplest examples of the moment map is the logarithm map that takes a point of the complex torus $\mathbb{C}^n$ to the point in $\mathbb{R}^n$ obtained by taking the logarithm of the absolute value coordinatewise. The images of holomorphic subvarieties of $\mathbb{C}^n$ under this map are called amoebas.

If one modifies this moment map by taking the logarithm with base $t$ and lets $t$ to go to infinity then the amoebas tend to some piecewise-linear polyhedral complexes in $\mathbb{R}^n$. The dimension of these limiting complexes is equal to the dimension of the original varieties. It turns out that such complexes can be considered as algebraic varieties over the so-called tropical semifield. The term "tropical semifield" appeared in Computer Science and, in the current context, refers to the real numbers augmented with the negative infinity and equipped with two operations, taking the maximum for addition and addition for multiplication.

Polynomials over the tropical semifield are convex piecewise-linear functions and geometric objects associated to these polynomials are certain piecewise-linear complexes in $\mathbb{R}^n$.

In the course we consider applications of both the amoebas themselves and the resulting tropical geometry. One area where amoebas turn out to be useful is Topology of Real Algebraic Varieties, in particular, problems related to Hilbert's 16th problem. Using amoebas we show topological uniqueness of a homologically maximal curve in the real torus $\mathbb{R}^n$ and deduce a partial topological description for hypersurfaces in $\mathbb{R}^n$ for $n > 2$. Applications of tropical geometry include construction of real algebraic varieties with prescribed topology (patchworking) as well as enumerative geometry.

A typical problem in enumerative algebraic geometry is to compute the number of curves of given degree and genus and with a given set of geometric constraints (e.g. passing through a point or another algebraic cycle, being tangent to such cycle, etc.). For a proper number of geometric constraints one expects a finite number of such curves. Even in the cases when this number is not finite there exists a way to interpret the answer to such problem as a (perhaps fractional or negative) Gromov-Witten number. Tropical geometry can be used for computation of these numbers. In this course we'll compute such numbers for arbitrary genus and degree when the ambient space is a toric surface and for genus 0 (and arbitrary degree) if the ambient space is a higher-dimensional toric variety.
In addition we consider real counterparts of the enumerative problems, in particular, the Welschinger invariant, and do some computations for them.

Prerequisite(s):
MAT 1194HF (MAT 449H1F) or some basic knowledge of Topology and Geometry. Contact the instructor for permission.

MAT 1197HF
GEOMETRY OF FLAG VARIETIES FOR SEMI-SIMPLE ALGEBRAIC GROUPS AND REPRESENTATION THEORY
S. Arkhipov

Below are the main topics to be covered in the course.
1. Recollections: Borel subgroups and maximal tori in a semi-simple algebraic group $G$, the weight and root lattices of a given maximal torus $T \subset G$, combinatorics of the root system for $T \subset B \subset G$, the Weyl group $W$ of $G$, the unipotent radical $U \subset B$.
3. The actions of $G$ on the set of Borel subgroups in $G$ and on the set of maximal unipotent subgroups in $G$. The flag variety $G/B$ and the base affine space $G/U$ for $G$.
10. Highest weight modules over the Lie algebra $\mathfrak{g}$. Verma and contragradient Verma modules.
12. Cotangent bundle to the Flag variety $T^*(G/B)$. Representation theory description. The nilpotent cone of $\mathfrak{g}$. Springer-Grothendieck map. Regular functions on $T^*(G/B)$ and on the nilpotent cone.
13. Conormal bundles to Schubert cells. Steinberg variety for $G$.
14. Convolution in Borel-Moore homology of the Steinberg variety.

Prerequisites.
The following topics are prerequisites for the course: minimal knowledge of finite dimensional Representation Theory for a semi-simple group $G$ (over complex numbers), singular homology and cohomology, basic Morse theory, a few definitions (like affine and projective varieties, line bundles etc.) from basic Algebraic Geometry.

MAT 1210HF
CLASS FIELD THEORY
H. Kim

This course is a continuation of MAT 1200H (Algebraic Number Theory). Consider the following

$$e^{\pi \sqrt{163}} = 262537412640768743.999999999999250072597...$$

The fact that $e^{\pi \sqrt{163}}$ is so close to an integer has a beautiful explanation that involves class field theory and theory of elliptic curves with complex multiplication. In particular, it involves the concept of Hilbert class field, the maximal unramified abelian expansion whose Galois group is the class group. In this course, we will prove, among other things, the existence of Hilbert class fields. We will also study Artin $L$-functions, and its
applications. If time permits, we will study explicit class field theory, namely, determining Hilbert class fields of imaginary quadratic fields using the theory of elliptic curves. We will use the modern exposition of using adeles and ideles.

Main references:
Algebraic Number Theory by S. Lang
Analytic Number Theory by L. Goldstein

Prerequisite: MAT 1200H (Algebraic Number Theory)

MAT 1303HF (CSC 2406HF)
TRIPLE SYSTEMS
E. Mendelsohn

Triple systems have their origins in algebraic geometry, statistics, and recreational mathematics. Although they are the simplest of combinatorial designs, the study of them encompasses most of the combinatorial, algebraic, and algorithmic techniques of combinatorial design theory.

Topics to be covered: Fundamentals of design; existence of triple systems; enumeration; computational methods; isomorphism and invariants; configuration theory; chromatic invariants; resolvability; directed, Mendelsohn, and mixed triples sytems.

Prerequisite(s):
Linear Algebra and an introductory course in combinatorics such as MAT 344HF are recommended.

MAT 1344HS
INTRODUCTION TO SYMPLECTIC GEOMETRY
Y. Karshon

We will discuss a variety of concepts, examples, and theorems of symplectic geometry and topology. These may include, but are not restricted to, these topics: review of differential forms and cohomology; symplectomorphisms; local normal forms; Hamiltonian mechanics; group actions and moment maps; geometric quantization; a glimpse of holomorphic techniques.

Prerequisites: manifolds, differential forms, cohomology.

Main recommended book:
Ana Cannas da Silva, "Lectures on Symplectic Geometry", Lecture Notes in Mathematics 1764, Springer-Verlag 2001. (A second edition should come out at some point but it's not out yet.)

Other relevant books:

MAT 1350HF
THE JONES POLYNOMIAL
D. Bar-Natan
The Jones polynomial is perhaps the simplest knot invariant to define; it can be defined (and will be defined in
the first class) in about 5 minutes, invariance can be proven in about 15 minutes, it can be programmed in another
10 minutes, and then it can be evaluated for the first few hundred knots in some 10 minutes or so. In the rest of
the semester we will see that the Jones polynomial has some knot theoretic implications, has lovely
generalizations and fits within some nice pictures, and is a wonderful excuse and unifying centre for the study of
several other deep subjects, including but not limited to combinatorics, homological algebra, Lie algebras,
quantum algebra, category theory and even quantum field theory. Some of these subjects we will cover in great
detail; others, for the luck of time, will only be briefly touched.

Prerequisites: Graduate core courses in Topology (MAT 1300Y) and in Algebra (MAT 1100Y), or their
equivalents.

MAT 1352HF
INDEX THEORY
E. Meinrenken

This course will be an introduction to the Atiyah-Singer index theorem for elliptic operators. We will cover the
following topics:
- Elliptic differential operators,
- K-theory
- The Atiyah-Singer index theorem
- Cohomological formulas
- Applications

Prerequisites:
Solid background knowledge in algebraic topology and manifold theory, as well as some Hilbert space basics.

References:
N. Higson, J. Roe: *Lectures on operator K-theory and the Atiyah-Singer index theorem*
D. Freed: *Notes on index theory*
G. Landweber: *K-theory and elliptic operators*

MAT 1355HF
RESOLUTION OF SINGULARITIES AND APPLICATIONS IN ANALYSIS AND IN GEOMETRY
P. Milman

In this course we will derive various applications of resolution of singularities to the classical-type inequalities of
Analysis. We will also examine some applications to problems in Geometry. The course will include a proof of a
mini variant of desingularization that will suffice for these applications.

Prerequisites: Standard undergraduate material (first 3 years) of the math specialist program including the
implicit function theorem (and related material) as well as some familiarity with the basic algebraic and
geometric notions such as polynomials, analytic functions, ideals, rings etc., affine and projective spaces.

MAT 1435HF
ANALYTIC METHODS IN COMBINATORICS
B. Szegedy
The goal of this course is to highlight some powerful analytic methods that prove to be extremely successful in modern combinatorics. A considerable part of the course will focus on the so called Regularity Lemma (by Szemeredi) and recent generalizations for hypergraphs (by Gowers, Rodl, Nagle, Skokan, Schacht, Kohayakawa). We will also discuss applications of harmonic analysis.

Prerequisites: Knowledge of graph theory and harmonic analysis.

**MAT 1450HS**
**SET THEORY: STRUCTURAL RAMSEY THEORY AND DYNAMICS OF GROUPS OF AUTOMORPHISMS**
S. Todorcevic

This course is a natural continuation of the 2005-06 MAT 1450HF course, but will not be bounded to those who took it as the subject matter is related but not too much dependent. The stress this time will be on finite Ramsey theory, or more precisely finite structural Ramsey theory, and its relationship to Fraisse theory of homogeneous structure and topological dynamics of groups of automorphisms.

Prerequisites: Basic core courses in mathematics.

Textbooks:
Graham-Rothschild-Spencer, Ramsey Theory, 1990
My textbook "Introduction to Ramsey Spaces", which at the moment is available as draft but at that time may be even in press.

**MAT 1502HS**
**STOCHASTIC CALCULUS**
J. Quastel


Prerequisite: Probability or Real analysis.

**MAT 1739HF**
**INTRODUCTION TO SUPERSYMMETRIC QUANTUM FIELD THEORIES**
K. Hori

This course will give an accessible introduction to supersymmetric quantum field theories in low dimensions. Supersymmetry is relevant for many recent developments in various fields in mathematics, including Gromov-Witten invariants, Donaldson invariants, Floer homology, various fixed point theorems, etc. No special background is assumed.

1. Gaussian integrals and Feynman diagrams, fermionic integrals.
2. Supersymmetric quantum mechanics and Morse theory.
3. Non-linear sigma models, Landau-Gizburg models, Linear sigma models.
4. Renormalization group flows, supersymmetric non-renormalization theorems.
5. Chiral rings and topological field theory.
Textbook:

References:

MAT 1844HF
INTRODUCTION TO SMOOTH DYNAMICS
C. Pugh

How ODE's integrate to flows, topological dynamics (minimality, topological transitivity, and related concepts), low dimensional dynamics such as Denjoy's Theorem for surface flows, hyperbolicity, linearization, stable manifold theory, structural stability, Axiom A dynamics and many examples, geodesic flows (especially with negative curvature), smooth ergodicity.

Prerequisites: The student should be at ease with respect to real analysis, differential topology, and linear algebra.

Books: Course notes will be based in part on the books Geometric Theory of Dynamical systems by Palis and deMelo, Invariant Manifolds by Hirsch, Pugh, and Shub, Global Stability of Dynamical Systems by Shub, Chaotic Dynamical Systems by Devaney, A First Course in Dynamics by Hasselblatt and Katok, Dynamical Systems by Robinson.

MAT 1845HS
CONFORMAL GEOMETRY AND HOLOMORPHIC DYNAMICS
M. Lyubich

Holomorphic dynamics studies iterates of rational endomorphisms of the Riemann sphere. Even in the simplest case of the quadratic family it is extremely rich. The tools of the field come from conformal and quasiconformal geometry, which have been, in turn, enriched by dynamics. We will discuss how these two fields interact and stimulate each other. The exposition will be self-contained.

MAT 1880HS
MATHEMATICAL METHODS IN BIOLOGY
I. M. Sigal

In this course we discuss two key groups of biological models which were intensively studied in the last few years. The first group deals with collective behaviour of interacting biological organisms such as cells and bacteria (e.g. chemotaxis). The goal here is to describe such phenomena as aggregation (congregation of cells or bacteria into tightly bound, rigid colonies) and developmental pattern formation.

The second group of models deals with mechanisms through which networks of interacting biomolecules (proteins or genes) carry out the essential functions in living cells. Among the questions which are addressed here is how the genetic and biochemical networks withstand considerable variations and random perturbations of biochemical parameters. The complexity and high inter-connectedness of these networks makes the question of the stability in their functioning of special importance.

Finally we will discuss mathematical models of the dynamics of HIV-1 and of cancer growth.
The models above are expressed in terms of Markov chains and stochastic ordinary differential equations. In addition, in the first case, reaction-diffusion equations (e.g. Keller-Segel equations) and stochastic particle dynamics are used. This mathematical background together with its biological interpretation will be developed in the course.

Prerequisites for this course: some familiarity with elementary ordinary and partial differential equations and elementary probability theory. No knowledge of biology is required.

MAT 1900Y/1901H/1902H
READINGS IN PURE MATHEMATICS

Numbers assigned for students wishing individual instruction in an area of pure mathematics.

MAT 1950Y/1951H/1952H
READINGS IN APPLIED MATHEMATICS

Numbers assigned for students wishing individual instruction in an area of applied mathematics.

STA 2111HF
GRADUATE PROBABILITY I
B. Virag

Random variables, expected value, independence, laws of large numbers, random walks, martingales, Markov chains

Prerequisite: Measure theory (may be taken at the same time) or permission of the instructor.

Textbook: Durrett, “Probability: Theory and Examples”

STA 2211HS
GRADUATE PROBABILITY II
J. Quastel

Weak convergence, central limit theorems, stable laws, infinitely divisible laws, ergodic theorems, Brownian motion

Textbook: Durrett, “Probability: Theory and Examples”

FIELDS INSTITUTE PROGRAM COURSE

MAT 1195HF
ABELIAN VARIETIES AND CRYPTOGRAPHY
K. Murty

The use of elliptic curves (one-dimensional Abelian varieties) in cryptography has been extensively studied over the last two decades. In this course, we will start to look at the higher-dimensional case. We will begin by discussing the necessary number theoretic and algebro-geometric background that will be necessary for this study. We will then discuss the case of elliptic curves and of Jacobians of various families of curves. Finally, we will discuss the case of general Abelian varieties. We will be looking at both constructive (i.e. explicit arithmetic and point counting) and destructive (i.e. weaknesses, attacks, etc.) aspects. The course will assume elementary number theory and abstract algebra.
COURSE IN TEACHING TECHNIQUES

The following course is offered to help train students to become effective tutorial leaders and eventually lecturers. It is not for degree credit and is not to be offered every year.

MAT 1499HS
TEACHING LARGE MATHEMATICS CLASSES
J. Repka

The goals of the course include techniques for teaching large classes, sensitivity to possible problems, and developing an ability to criticize one's own teaching and correct problems.

Assignments will include such things as preparing sample classes, tests, assignments, course outlines, designs for new courses, instructions for teaching assistants, identifying and dealing with various types of problems, dealing with administrative requirements, etc.

The course will also include teaching a few classes in a large course under the supervision of the instructor. A video camera will be available to enable students to tape their teaching for later (private) assessment.

COURSES FOR GRADUATE STUDENTS FROM OTHER DEPARTMENTS

(Math graduate students cannot take the following courses for graduate credit.)

MAT 2000Y READINGS IN THEORETICAL MATHEMATICS
MAT 2001H READINGS IN THEORETICAL MATHEMATICS I
MAT 2002H READINGS IN THEORETICAL MATHEMATICS II

(These courses are used as reading courses for engineering and science students in need of instruction in special topics in theoretical mathematics. These course numbers can also be used as dual numbers for some third and fourth year undergraduate mathematics courses if the instructor agrees to adapt the courses to the special needs of graduate students. A listing of such courses is available in the 2006-2007 Faculty of Arts and Science Calendar. Students taking these courses should get an enrolment form from the graduate studies office of the Mathematics Department. Permission from the instructor is required.)

5. RESEARCH ACTIVITIES

The Department of Mathematics offers numerous research activities, in which graduate students are encouraged to participate. Research seminars are organized informally at the beginning of each year by one or more faculty members, and are offered to faculty and graduate students on a weekly basis throughout the year. The level and specific content of these seminars varies from year to year, depending upon current faculty and student interest, and upon the availability and interests of invited guest lecturers. The following weekly research seminars have been offered in the past few years:

**Research Seminars**

- Applied Math/PDE/Analysis/Seminar
- Probability Seminar
- I-AIM Interdisciplinary Math Seminar
- Dynamical Systems Seminar
- Computability and Complexity in Analysis and Dynamics Seminar
- Geometry and Topology Seminar
- Symplectic Seminar
6. ADMISSION REQUIREMENTS AND APPLICATION PROCEDURES

Due to the large numbers of applications received in the Department of Mathematics each year, serious consideration will only be given to applicants with strong backgrounds in theoretical mathematics and with first class academic standing.

Application materials and admission requirements are available from the Department of Mathematics web-site: http://www.math.toronto.edu/graduate

Please read all instructions carefully and note the deadlines. In addition, the Department of Mathematics requires three letters of reference. The letters must be from three people familiar with your mathematical work, giving their assessment of your potential for graduate study and research in mathematics.

It is essential that all incoming graduate students have a good command of English. Facility in the English language must be demonstrated by all applicants educated outside Canada whose primary language is not English. This requirement is a condition of admission and should be met before application. There are three ways to satisfy this requirement: (1) Test of English as a Foreign Language (TOEFL): (a) internet-based test (iBT), minimum score of 22/30 for both the Writing and Speaking sections, with an overall minimum TOEFL score of 93/120, or (b) computer-based test, minimum score 237, with Essay Rating, minimum score 4.0, or (c) paper-based test, minimum score 580, with TWE (Test of Written English), minimum score 4.0; (2) a score of at least 85 on the Michigan English Language Assessment Batter (MELAB); (3) a score of at least 7.0 on the International English Language Testing Service (IELTS). Applicants are required to satisfy this requirement by February (for September admission) so that scores are available at the time applications are considered.

7. FEES AND FINANCIAL ASSISTANCE

Fees

Information concerning fees is in the admissions package that accompanies the application forms; more detailed information and a schedule of fees for 2006-07 will be available in August 2006 from the School of Graduate Studies. Listed below are the fees for the 2006-07 academic session, including incidental fees and the health
insurance premium for visa students:

<table>
<thead>
<tr>
<th>Student Type</th>
<th>Status</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Domestic students</td>
<td>Full-time</td>
<td>$6,629</td>
</tr>
<tr>
<td>Visa students</td>
<td>Full-time</td>
<td>$14,253</td>
</tr>
</tbody>
</table>

**Financial Assistance**

Below is a list of those types of financial assistance most commonly awarded to mathematics graduate students in 2006-07. This information should also be applicable for students who wish to apply for the 2007-08 academic year; the deadlines for applications will be altered slightly in accordance with the 2007-08 academic year. Some awards are available from external funding agencies; others come from within the University.

Less common scholarships, offered by smaller or foreign funding agencies, are also available; information about these is generally distributed to the graduate offices of local universities.

**Natural Sciences and Engineering Research Council (NSERC) Postgraduate Scholarships and Canada Graduate Scholarships**

*Value*: approx. $17,300-$35,000 for a twelve month period

*Eligibility*: Canadian citizens, permanent residents; first class academic standing; full-time attendance (restricted almost exclusively to Canadian universities)

*Application*: apply through the university you are currently attending; application available at [www.nserc.ca](http://www.nserc.ca)

*Deadline*: around mid October. Contact your university for departmental deadline

**Ontario Graduate Scholarships (OGS)**

*Value*: approx. $5,000 per term for two or three terms

*Eligibility*: no citizenship restrictions; first class academic standing; full-time attendance at an Ontario university

*Application*: apply through the university you are currently attending or contact the OGS office in Thunder Bay, Ontario directly at 1-800-465-3957 ([http://osap.gov.on.ca/eng/eng_osap_main.html](http://osap.gov.on.ca/eng/eng_osap_main.html)).

*Deadline*: around mid October

**University of Toronto Fellowships**

*Value*: minimum $1,000

*Eligibility*: no citizenship restrictions; at least an A- average; full-time attendance at the University of Toronto (or, in the case of the Master’s program and for students with a disability, part-time registration together with a letter from the Director of Special Services to Persons with a Disability confirming that part-time study is de facto full-time study for the student). See below for policy.

*Application*: graduate school applicants will be considered automatically

*Deadline*: February 1

**Connaught Scholarship**

*Value*: $15,000 plus full fees for first year

*Eligibility*: no citizenship restrictions; full-time attendance at the University of Toronto; awarded only to exceptionally outstanding scholars

*Application*: graduate school applicants will be considered automatically

*Deadline*: February 1

**Research Assistantships**

*Value*: a limited amount of funds is available for academically worthy students

*Eligibility*: no citizenship restrictions; full-time attendance; high academic standing

*Application*: graduate school applicants will be considered automatically

*Deadline*: February 1

**Teaching Assistantships**

*Value*: approx. $27-$30 per hour; number of hours per week will not exceed a maximum average of 8
Eligibility: all full-time students who are accepted by the Mathematics Department (subject to satisfactory performance); may be held in conjunction with other awards

Application: forms available in May from the Graduate Office, Department of Mathematics

Deadline: June 20

Department of Mathematics Policy on Financial Support of Graduate Students

Ph.D. Students: At the time of admission to the Ph.D. program, students will normally be guaranteed support for a period of four years (five years in the case of students admitted directly from a Bachelor’s program), except that students who complete their degree requirements earlier will not be supported past the end of the academic year in which they finish. This guarantee will be made up of a mix of fellowships (including external awards such as NSERC), teaching assistantships and other sources of funding, at the discretion of the Department; and is subject to satisfactory academic progress, the maintenance of good standing, and in the case of teaching assistantships, satisfactory performance in that role, as judged by the Department. Absent this, support may be reduced, suspended, or discontinued.

In exceptional circumstances some funding may be provided to students in a subsequent year, but the Department expects that students will normally have completed their degree requirements within the four year period.

M.Sc. Students: Students who are granted provisional admission to the Ph.D. program at the time of admission will receive financial support, normally for one year only.

All full-time students in the first or second year of a Master’s program are eligible for teaching assistant work (subject to satisfactory performance).

8. OTHER INFORMATION

The Department of Mathematics is located in the heart of the University of Toronto, which in turn is located in the heart of downtown Toronto. Students therefore have access to a wide range of facilities and services. A list of these appear below.

Facilities and Services

Library Facilities
The University of Toronto Library System is the 4th largest academic research library in North America. It has over 4 million volumes. The Mathematical Sciences Library (MSL) is in the same building as the Mathematics Department. The vast majority of Mathematics journals are held at the MSL with some being held at the GSIC. The MSL also houses a selective collection of over 17,000 titles of mathematical books. The resources of the Library System are electronically linked by UTLink. It includes the catalogues of all libraries on campus, electronic abstracts and indexes, and other electronic resources such as electronic journals and internet. The MSL has computers with Netscape which give access to MathSciNet, UTLink and other electronic resources. It has study spaces for reading journals/books and studying. The Library has a photocopier for copying library materials. The MSL gives each graduate student a photocopying allowance. The Gerstein Science Information Centre (GSIC) also has a comprehensive collection of mathematical books up to 1998.

Computer Facilities
Every graduate student may obtain an account on the computer system. The Department of Mathematics currently has an IBM RS6000 machine running AIX5L as the main server, and an AMD machine running Linux as a compute server. The fastest of these, for floating point computations, is currently the dual processor IBM 44P-270. The AMD machine has better integer performance. These servers can support multiple users through several X-terminals, and ascii-terminals. There are a few PC's available. The main departmental location, in Bahen, has wireless network coverage maintained by the University. Programming facilities include C, Fortran, Maple, Mathematica and TeX (LaTeX, AMS-TeX, AMS-LaTeX).
Housing
The university operates five graduate student residences-apartment complexes on or near the campus, ranging from unfurnished family apartments to the more conventional bed-and-board residences. In addition, the University Housing Service provides a listing of privately owned rooms, apartments and houses available for students to rent (see Contacts below). More detailed information about these facilities may be found in the admissions package which accompanies the application forms.

Students should keep in mind that accommodation could be expensive and limited, particularly in downtown Toronto. It is therefore advisable to make inquiries well in advance and to arrive in Toronto a few days prior to the start of term. Students can expect to pay anywhere between $500 to $900 per month on accommodation and from $300 to $500 per month on food, travel and household necessities.

Health Services
The University of Toronto Health Service offers medical services and referrals to private physicians for University of Toronto students. Most of these services are free of charge if you are covered under Ontario Health Coverage (OHIP), or the University Health Insurance Plan (UHIP) for visa students. OHIP application forms and information are available from the University Health Services (see Contacts below). UHIP coverage for visa students is compulsory and is arranged during registration at the International Student Centre.

Disabled Students
Services and facilities for disabled students are available at the University of Toronto. Further information may be obtained from the Coordinator of Services for Disabled Persons (see Contacts section below).

Foreign Students
The International Student Centre offers many services to foreign students, including an orientation program in late August–early September, individual counselling whenever appropriate, and an English language program. In addition, the International Student Centre contacts all foreign students once they have been accepted into the graduate program, to provide information and advice concerning immigration procedures (visa and student authorization forms), employment restrictions and authorization while in Canada, and other relevant matters (see Contacts below).

Athletics & Recreation
A wide range of athletic facilities are available within the university, including an arena and stadium, playing fields, swimming pools, squash, tennis, badminton, volleyball and basketball courts, running tracks, archery and golf ranges, fencing salons, exercise and wrestling rooms, dance studios, saunas, lockers and a sports store. Instruction courses, exercise classes and fitness testing are regularly offered, and there is an extensive intramural program with several levels of competition in more than 30 sports.

Other recreational activities and facilities are also available within the university, such as theatre, music, pubs, dances, art exhibitions, a wide range of clubs, debates lectures and seminars, reading rooms, cafeterias and chapels.

University of Toronto students also enjoy easy access (walking distance or only a few minutes by subway) to symphony concerts, theatres, ballet, operas, movies, restaurants and shopping.

Graduate Student Associations
Every graduate student at the University of Toronto is automatically a member of the Graduate Student Union (GSU). Graduate students in the Department of Mathematics are also members of the Mathematical Graduate Students Association (MGSA). Between them, these associations sponsor many events every year, including parties, pubs, dances, outings and more serious endeavours such as seminars and lectures.

Contacts and Sources of Information

Graduate Office
Department of Mathematics
University of Toronto
40 St. George St., Room 6291
Toronto, Ontario M5S 3G3
(416) 978-7894
(416) 978-4107 (Fax)
ida@math.toronto.edu
http://www.math.toronto.edu/graduate
- all matters relating to graduate studies in mathematics at the University of Toronto
- location of registration in September/January

Mathematics Library
University of Toronto
40 St. George Street, Room 6141
Toronto, Ontario M5S 3G3
(416) 978-8624
(416) 978-4107 (Fax)
mathlib@math.toronto.edu

School of Graduate Studies
University of Toronto
63 St. George Street
Toronto, Ontario
Canada M5S 2Z9
(416) 978-5369
(416) 978-4367 (Fax)
graduate.information@utoronto.ca
http://www.sgs.utoronto.ca
- general information concerning graduate studies at the University of Toronto

Fees Department
Office of the Comptroller
University of Toronto
215 Huron Street, 3rd Floor
Toronto, Ontario M5S 1A1
(416) 978-2142
(416) 978-2610 (Fax)
fees@finance.utoronto.ca
http://www.students.utoronto.ca/Money_Money_Money/Tuition_and_Fees.htm
- enquiries concerning fees
- payment of fees

Special Services to Persons with a Disability
University of Toronto
214 College Street, 1st Floor
Toronto, Ontario M5T 2Z9
(416) 978-8060
(416) 978-8246
http://www.library.utoronto.ca/www/equity/ssd.htm
- facilitates the inclusion of students with hidden or obvious disabilities and health conditions into university life

University Housing Service
University of Toronto
214 College Street, 1st Floor
Toronto, Ontario M5T 2Z9
416-978-8045
416-978-1616 (Fax)
housing.services@utoronto.ca
http://link.library.utoronto.ca/StudentHousing/
• information concerning university residences and rental accommodations

University Health Service
University of Toronto
214 College Street, 2nd Floor
Toronto, Ontario M5T 2Z9
416-978-8030
416-978-2089 (Fax)
http://www.utoronto.ca/health/
• medical assistance for University of Toronto students
• application forms for Ontario Health Coverage

International Student Centre
University of Toronto
33 St. George Street
Toronto, Ontario M5S 2E3
(416) 978-2564
(416) 978-4090 (Fax)
isc.information@utoronto.ca
http://www.library.utoronto.ca/www/isc/
• information and assistance for international students, including UHIP registration

The Athletic Centre
University of Toronto
55 Harbord Street
Toronto, Ontario M5S 2W6
(416) 978-3437
416-978-6978 (Fax)
http://www.ac-fpeh.com/
• athletics and recreation information

Hart House
University of Toronto
7 Hart House Circle
Toronto, Ontario M5S 3H3
416-978-4411
colin.furness@utoronto.ca
http://www.utoronto.ca/harthouse
• athletics and recreation information

Graduate Students’ Union
University of Toronto
16 Bancroft Avenue
Toronto, Ontario M5S 1C1
416-978-2391 or -6233
416-971-2362 (Fax)
gsunion@chass.utoronto.ca
http://www.utoronto.ca/gsunion/

Sexual Harassment Office
University of Toronto
40 Sussex Avenue
Students are covered by the Sexual Harassment Policy while on university premises or carrying on a university-related activity. Complaints and requests for information are confidential.

Human Resources Development Canada (HRDC)
811 Danforth Avenue, 1st Floor, or
25 St. Clair Avenue East, 1st Floor
1-800-206-7218

- To obtain a Social Insurance Number (in person only). Office hours: Monday-Friday, 08:30-16:00
- Applications available from [http://www.hrdc-drhc.gc.ca/sin-nas/0300/0300_000_e.shtml](http://www.hrdc-drhc.gc.ca/sin-nas/0300/0300_000_e.shtml). Supporting documentation must be original, e.g. student authorization and an offer of employment letter
- Takes an average of 4 weeks to process

APPENDIX A: COMPREHENSIVE EXAMINATION SYLLABI

Algebra
1. Linear algebra. Students will be expected to have a good grounding in linear algebra, vector spaces, dual spaces, direct sum, linear transformations and matrices, determinants, eigenvectors, minimal polynomials, Jordan canonical form, Cayley-Hamilton theorem, symmetric, alternating and Hermitian forms, polar decomosition.
2. Group Theory. Isomorphism theorems, group actions, Jordan-Hölder theorem, Sylow theorems, direct and semidirect products, finitely generated abelian groups, simple groups, symmetric groups, linear groups, nilpotent and solvable groups, generators and relations.
4. Modules. Modules and algebras over a ring, tensor products, modules over a principal ideal domain, applications to linear algebra, structure of semisimple algebras, application to representation theory of finite groups.
5. Fields. Algebraic and transcendental extensions, normal and separable extensions, fundamental theorem of Galois theory, solution of equations by radicals.

No reference is provided for the linear algebra material.

References for the other material:
Dummit & Foote: *Abstract Algebra*, Chapters 1-14 (pp. 17-568).
Alperin & Bell: *Groups and Representations*, Chapter 2 (pp. 39-62), 5, 6 (pp. 107-178).

Complex Analysis
3. Conformal mapping, Riemann mapping theorem.

Note: The material in Ahlfors can largely be replaced by Chapters 10, 11, 12.1-12.6, and 14 of Rudin. But Ahlfors is the official syllabus for this material. The second edition of Ahlfors can be used if it is noted that Section 5.5 in the third edition is Section 5.4 in the second edition.)

Real Analysis
References:

1. Background: Royden, Chapters 1 and 2; Folland (Prologue).
2. Basic Measure Theory: Royden, Chapters 3 and 4, for the classical case on the real line (which contains all the basic ideas and essential difficulties), then Chapter 11, Sections 1-4, for the general abstract case; Folland, Chapters 1 and 2.
3. Differentiation: Royden, Chapter 5, for the classical case, then Chapter 11, Sections 5 and 6 for the general case; Folland, Chapter 3 (For differentiation on $\mathbb{R}^n$ one can restrict the attention to the one dimensional case, which contains all the basic ideas and essential difficulties.)
4. Basic Functional Analysis: Royden, Chapter 10, Sections 1, 2, 3, 4, 8; Folland, Chapter 5, Sections 1, 2, 3, 5.
5. $L^p$-Spaces: Royden, Chapter 6 for the classical case, and Chapter 11, Section 7 for the general case, Chapter 13, Section 5 for the Riesz Representation Theorem; Folland, Chapter 6, Sections 1 and 2, Chapter 7, Section 1.
6. Harmonic Analysis: Katznelson, Chapter 1, Chapter 2, Sections 1 and 2, and Chapter 6, Sections 1 to 4; Folland, Chapter 8, Sections 1, 2, 3, 4, 5, and 8. One can restrict the attention to the one dimensional case, as done in Katznelson.

**Topology**


**Partial Differential Equations**

Note: This is meant to be an exam syllabus not a course outline.

As such, topics are not necessarily ordered as in a logical development.

1) **Basic Notions in Ordinary Differential Equations**: Fundamental theorem on existence and uniqueness of solutions of $y' = f(x,y)$ when $f$ is Lipschitz w.r.t. $y$. Fixed point theorem, Picard iterates. (Various topics in PDE will also assume familiarity with undergraduate ODE material.)

2) **Basic Notions in Linear Partial Differential Equations**
   b) **Parabolic PDEs**: Heat Equation, fundamental solution of the heat equation, mean value property, maximum principle, regularity properties, initial value problem for the heat equation, semigroups, gradient flows
c) **Hyperbolic PDEs**: Wave equation, fundamental solution of the wave equation, spherical means, Huygen's principle, conservation of energy, finite speed of propagation, initial value problem for the wave equation, other hyperbolic PDEs.

3) **Distributions; Fourier Transform**

4) **Sobolev spaces; Weak Solutions**: Weak derivatives, Sobolev spaces $W^{k,p}$, $L^p$ based fractional Sobolev spaces $H^s$, Approximation properties, Extensions, Traces, Sobolev inequalities, Poincaré lemma. 

**Weak solutions and regularity theory is enmeshed with the topics on the exam.**

5) **Nonlinear PDEs**: First-Order: Method of characteristics, Hamilton-Jacobi equations, Conservation laws, weak solutions, shocks and rarefactions, uniqueness and entropy solutions. 

**Second-Order**: gradient flows, linearization around special solutions, vanishing viscosity limit of Burger's equation.

6) **Calculus of variations**: direct methods, convexity, weak-* continuity and compactness, first and second variations, Euler-Lagrange equation, Lagrange multipliers, constraints

**References**:

V. I. Arnold: *Ordinary differential equations* 1992
W. Hurewicz: Lectures on ordinary differential equations 1990

**APPENDIX B: APPLIED MATHEMATICS COMPREHENSIVE EXAMINATION SYLLABI**

A student planning to specialize in applied mathematics must pass three comprehensive exams, at least two of which are a general written exam (algebra, analysis (real and complex), topology, or partial differential equations (PDE I and PDE II)). The following are samples of other exam topics.

**Combinatorics**


2) **Graph Theory**: Trees, connectivity, bipartite graphs, minimal spanning trees, Eulerian and Hamiltonian graphs, travelling salesman and chinese postman problems, matchings, chromatic number, perfect graphs.

3) **Design Theory**: Definitions, examples, finite fields, finite affine and projective spaces, Fisher’s inequality, symmetric designs, statement of Wilson’s Theorem and Wilson’s Fundamental Construction.

4) **Coding Theory**: Linear codes, sphere packing, Hamming and Plotkin bounds, perfect codes, polynomial over finite fields.

5) **Algorithms and Complexity**: Algorithms for listing permutations, combinations, subsets. Analysis of algorithms, basic concepts such as NP, and #P, and NPC.

**References**:


**Control Theory and Optimization**

1) **Control Theory**: Qualitative properties of the reachable sets, Lie bracket and Lie determined systems, linear theory, stability and feedback. *(Reference: V. Jurdjevic: Geometric Control Theory, Cambridge University Press, Chapters 1,2 and 3)*

2) **Optimal Control**: Linear-quadratic problems, symplectic form, Lagrangians, the Riccati equation, the Maximum Principle and its relation to the calculus of variations. *(Reference: V. Jurdjevic: Geometric Control Theory, Cambridge University Press, Chapters 7, 8,11)*

3) **Linear Programming**: Convex analysis, simplex algorithm, duality, computational complexity and Karmarkar’s Algorithm. *(Reference: Bazaraa, Jarvis & Sherali: Linear Programming and Network Flows, Wiley, 1990, Chapters 2,3,4,6,8)*

4) **Nonlinear Programming**: Unconstrained and constrained nonlinear problems. Introduction to computational methods. *(Reference: Luenberger: Linear and Nonlinear Programming, Addison-Wesley, 1984, Chapters 6,7,10)*
**Fluid Mechanics**
It is expected that a student has a basic knowledge of real and complex analysis including ordinary differential equations. The extra mathematics required includes:

1) **Partial differential equations**: Laplace’s equation, properties of harmonic functions, potential theory, heat equation, wave equation. Solutions through series and transform techniques. Bessel functions and Legendre functions. Distributions. (e.g. Duff and Naylor)

2) **Asymptotic and perturbation techniques**: Asymptotic series solutions of ordinary differential equations, asymptotic expansion of integrals. Singular perturbation problems, boundary layer methods, WKB theory, multiple time-scale analysis. (e.g. Bender and Orszag)

Basic physical properties of fluids. Derivation of the Navier-Stokes equations for a viscous compressible fluid; vorticity; energy balance. Simple exact solutions of the Navier-Stokes equations. Slow viscous motions; Stokes flows; Oseen flow. Irrotational flow; sources and sinks; complex variable methods. Boundary layer approximation. Blasius flow; separation; jets and wakes. Rotating flows; geostrophic behaviour. Free surface flows; wave propagation. Simple unsteady boundary layer flows; Stokes layers. Shock waves in a tube; supersonic flow.

**General Relativity and Classical Mechanics**

1) Space-times as Lorentz manifolds. Differential geometry (curvature, etc.) and local and global properties of Lorentz manifolds.
2) Field equations of general relativity, stationary and static space-times. Exact solutions. Schwarzschild, Kruskal, Kerr solutions. Cosmological models: Robertson-Walker and Friedman models and their properties.
3) Cauchy problem for the field equations. Classification of space-times.
4) Symplectic geometry, symplectic structure of cotangent bundles, Poisson brackets.
5) Hamiltonian equations, canonical transformations, Legendre transformations, Lagrangian systems, Hamilton-Jacobi theory.

**References**
Hawking & Ellis: *Large Scale Structure of Space-Time*, Chapters 2,3,4,5.
O’Neill: *Semi-Riemannian Geometry with Applications to Relativity*.
Wald: *General Relativity*, Chapters 1-6 and Appendices A-C and E

**Mathematical Finance**

1) **Stochastic calculus**: Martingales, Ito’s lemma, Girsanov’s theorem, stochastic differential equations, stopping times. *(Reference: Baxter & Rennie, *Financial Calculus*)

**Probability**
The Probability Exam is a written exam and is administered by the Department of Statistics. It is based on material covered in STA 2111F and STA 2211S.
Topics covered include:

1) **Elementary probability theory**: Bernoulli trails, combinatorics, properties of standard probability distributions, Poisson processes, Markov chains
2) **Probability spaces**: measure theory and Lebesgue integration, extension theorems, Borel-Cantellis lemmas, product measures and independence, Fubini’s Theorem
3) **Random variables and expectations**: probability distributions, Radon-Nikodym derivatives and densities, convergence theorems such as dominated convergence, monotone convergence, etc
4) **Limit theorems**: inequalities, weak and strong laws of large numbers for sums of i.i.d. random variables, Glivenko-Cantelli Theorem, weak convergence (convergence in distribution), continuity theorem for characteristic functions, Central Limit theorems
5) **Conditional probability and expectation**: definitions and properties, statistical applications, martingales
6) **Basics of Brownian motion and diffusions**

**References**
Most of the above material is covered in any one of the following texts:

- P. Billingsley: *Probability and Measure*, 1995
- L. Breiman: *Probability*, 1992

### Quantum Field Theory


3) **Relativistic quantum field theory**: The Klein-Gordon and Dirac equations. Fock space for spin 0 and spin \( \frac{1}{2} \) particles. Creation and annihilation operators. Quantum fields as operator-valued distributions. The Gupta-Bleuler formalism for photons and gauge freedom.

References:

**Intermediate:**

**Advanced:**

### APPENDIX C: PH.D. DEGREES CONFERRED FROM 1994-2006

**1994**

BODNAR, Andrea (Fluid Mechanics) *Low Reynolds Number Particle-Fluid Interactions*

FERRANDO, Sebastian (Ergodic Theory) *Moving Convergence for Superadditive Processes and Hilbert Transform*

FRY, Robb (Functional Analysis) *Approximation on Banach Spaces*

HA, Minh Dzung (Ergodic Theory) *Operators with Gaussian Distributions, \( L^2 \) -Entropy and a.e. Convergence*

HA, Xianwei (Representation Theory) *Invariant Measure on Sums of Symmetric Matrices and its Singularities and Zero Points*

ŁABA, Izabella (Mathematical Physics) *N-Particle Scattering in a Constant Magnetic Field*

LISI, Carlo (Operator Theory) *Perturbation by Rank-Two Projections*

MA, Kenneth (Fluid Mechanics) *Low Reynolds Number Flow in the Presence of a Corrugated Boundary*
PRANOTO, Iwan (Control Theory) Distributed Parameter System: Controllability and its Related Problems

STEVENS, Ken (Operator Algebras) The Classification of Certain Non-Simple Approximate Interval Algebras

1995
GUZMAN-GOMEZ, Marisela (PDE) Regularity Properties of the Davey-Stewartson System for Gravity-Capillary Waves
LI, Lianqing (Operator Algebras) Classification of Simple C*-Algebras: Inductive Limit of Matrix Algebras over 1-Dimensional Space
LOUKANIDIS, Dimitrios (Algebraic Groups) Bounded Generation of Certain Chevalley Groups
MONROY-PEREZ, Felipe (Control Theory) Non-Euclidean Dubins' Problem: A Control Theoretic Approach
PYKE, Randall (PDE) Time Periodic Solutions of Nonlinear Wave Equations
STANLEY, Catherine (Algebra) The Decomposition of Automorphisms of Modules over Rings
TIE, Jingzhi (PDE) Analysis on the Heisenberg Group and Applications to the $\tilde{\partial}$-Neumann Problem

1996
DEPAEPE, Karl (Lie Algebras) Primitive Effective Pairs of Lie Algebras
JUNQUEIRA, Lucia (Set Theory) Preservation of Topological Properties by Forcing and by Elementary Submodels
SCHANZ, Ulrich (PDE) On the Evolution of Gravity-Capillary Waves in Three Dimension

1997
AKBARY-MAJDAKADNO, Amir (Number Theory) Non-Vanishing of Modular L-functions with Large Level
AUSTIN, Peter (Group Theory) Products of Involutions in the Chevalley Groups of Type $F_4(K)$
CLOAD, Bruce (Operator Theory) Commutants of Composition Operators
CUNNINGHAM, Clifton (Automorphic Forms) Characters of Depth-Zero Supercuspidal Representations of $Sp_4(F)$: From Perverse Sheaves to Shalika Germs
DOOLITTLE, Edward (PDE) A Parametrix for Stable Step Two Hypoelliptic Partial Differential Operators
FARAH, Ilijas (Set Theory) Analytic Ideals and their Quotients
HOMAYOUNI-BOROOJENI, Soheil (Set Theoretic Topology) Partition Calculus for Topological Spaces
STANLEY, Donald (Algebraic Topology) Closed Model Categories and Monoidal Categories
SUN, Heng (Automorphic Forms) The Residual Spectrum of $\overline{GL(N)}$: The Borel Case
THERIAULT, Stephen (Algebraic Topology) A Reconstruction of Anick's Fibration $S^{2n-1} \to T^{2n-1}(p^r) \to \Omega S^{2n+1}$

1998
BALLANTINE, Cristina (Automorphic Forms) Hypergraphs and Automorphic Forms
CENTORE, Paul (Differential Geometry) A Mean-Value Laplacian for Finsler Spaces
CHEN, Qun (Algebraic Geometry) Hilbert-Kunz Multiplicity of Plane Curves and a Conjecture of K. Pardue
GRUNBERG ALMEIDA PRADO, Renata (Set-theoretic Topology) Applications of Reflection to Topology
HILL, Peter (Knot Theory) On Double-Torus Knots
LI, Mingchu (Combinatorics) Hamiltonian Properties of Claw-Free Graphs
MEZO, Paul (Automorphic Forms) A Global Comparison for General Linear Groups and their Metaplectic Coverings
STEVENS, Brett (Combinatorics) Transversal Covers and Packings
STEVENS, Irina (C*-Algebras) Hereditary Subalgebras of AI Algebras
TRAVES, William (Algebraic Geometry) Differential Operators and Nakai's Conjecture

1999
DEAN, Andrew (C*-Algebras) A Continuous Field of Projectionless C*-Algebras
FRIS, Peter (C*-Algebras) Normal Elements with Finite Spectrum in C*-Algebras of Real Rank Zero
GUSTAFSON, Stephen (Mathematical Physics) Some Mathematical Problems in the Ginzburg-Landau Theory of Superconductivity
MAHMOUDIAN, Kambiz (Number Theory) A Non-Abelian Analogue of the Least Character Nonresidue
MAHVIDI, Ali (Operator Theory) Invariant Subspaces of Composition Operators

36
SCHIPERS, Eric (Complex Variables) *The Calculus of Conformal Metrics and Univalence Criteria for Holomorphic Functions*

YANG, Q. (Lie Algebras) *Some Graded Lie Algebra Structures Associated with Lie Algebras and Lie Algebroids*

### 2000

CALIN, Ovidiu (Differential Geometry) *The Missing Direction and Differential Geometry on Heisenberg Manifolds*

DERANGO, Alessandro (C*-Algebras) *On C*-Algebras Associated with Homeomorphisms of the Unit Circle*

HIRSCORN, James (Set Theory) *Cohen and Random Reals*

MADORE, Blair (Ergodic Theory) *Rank One Group Actions with Simple Mixing Z Subactions*

MARTINEZ-AVENDANO, Rubén (Operator Theory) *Hankel Operators and Generalizations*

MIGHTON, John (Knot Theory) *Topics in Ramsey Theory of Sets of Real Numbers*

MORREY, Justin (Set Theory) *Topics in Ramsey Theory of Sets of Real Numbers*

RAZAK, Shaloub (C*-Algebras) *Classification of Simple Stably Projectionless C*-Algebras*

SCOTT, Jonathan (Algebraic Topology) *Algebraic Structure in Loop Space Homology*

ZHAN, Yi (PDE) *Viscosity Solution Theory of Nonlinear Degenerate*

### 2001

COLEMAN, James (Nonlinear PDE's) *Blowup Phenomena for the Vector Nonlinear Schrödinger Equation*

IZADI, Farz-Ali (Differential Geometry) *Rectification of Circles, Spheres, and Classical Geometries*

KERR, David (C*-Algebras) *Pressure for Automorphisms of Exact C*-Algebras and a Non-Commutative Variational Principle*

OLWA, Chris (Mathematical Physics) *Some Mathematical Problems in Inhomogeneous Cosmology*

PIVATO, Marcus (Mathematical Finance) *Analytical Methods for Multivariate Stable Probability Distributions*

POON, Edward (Operator Theory) *Frames of Orthogonal Projections*

SAUNDERS, David (Mathematical Finance) *Mathematical Problems in the Theory of Incomplete Markets*

SOLTYSS-KULINICZ, Michael (Complexity) *The Complexity of Derivations of Matrix Identities*

VASILIEV, Branislav (Mathematical Physics) *Mathematical Theory of Tunneling at Positive Temperatures*

YUEN, Waikong (Probability) *Application of Geometric Bounds to Convergence Rates of Markov Chains and Markov Processes on R^n*

### 2002

HERNANDEZ-PEREZ, Nicholas (Math. Finance) *Applications of Descriptive Measures in Risk Management*

KAVEH, Kiumars (Algebraic Geometry) *Morse Theory and Euler Characteristic of Sections of Spherical Varieties*

MOHAMMADALIKANI, Ramin (Symplectic Geometry) *Cohomology Ring of Symplectic Reductions*

SOPROUNOVA, Eugenia (Algebraic Geometry) *Zeros of Systems of Exponential Sums and Trigonometric Polynomials*

THERIAULT, Nicolas (Algebraic Number Theory) *The discrete logarithm problem in the Jacobian of algebraic curves*

### 2003

ADAMUS, Janus (Analytic Geometry) *Vertical components in fibre powers of analytic mappings*

BUBENIK, Peter (Algebraic Topology) *Cell attachments and the homology of loop spaces and differential graded algebras*

HO, Nan-Kuo (Symplectic Geometry) *The moduli space of gauge equivalence classes of flat connections over a compact nonorientable surface*

JONG, Peter (Ergodic Theory) *On the Isomorphism Problem of p-Endomorphisms*

PEREIRA, Rajesh (Operator Theory) *Trace Vectors in Matrix Analysis*

STAUBACH, Wolfgang (PDE) *Path Integrals, Microlocal Analysis and the Fundamental Solution for Hörmander Laplacians*

THERIAULT, Nicolas (Algebraic Number Theory) *The discrete logarithm problem in the Jacobian of algebraic curves*

TING, Fridolin (Mathematical Physics) *Pinning of magnetic vortices by external potential*
TSANG, Kin Wai (Operator Algebras) *A Classification of Certain Simple Stably Projectionless C*-Algebras*

2004
AHMAD, Najma (Applied Math) *The geometry of shape recognition via the Monge-Kantorovich optimal transportation problem* (in conjunction with Brown University)
BRANKER, Maritza (Several Complex Variables) *Weighted approximation in \( \mathbb{R}^n \*)
CHEN, Oliver (Mathematical Finance) *Credit barrier models*
ESCOBAR ANEL, Marcos (Mathematical Finance) *Mathematical treatment of commodity markets*
HUNG, Ching-Nam (Operator Algebras) *The numerical range and the core of Hilbert-space operators*
IVANESCU, Cristian (Operator Algebras) *On the classification of simple C*-algebras which are inductive limits of continuous-trace C*-algebras with spectrum the closed interval [0,1]*
KIRITCHENKO, Valentina (Analytic Geometry) *A Gauss-Bonnet Theorem, Chern Classes and an Adjunction Formula for Reductive Groups*
KUZNETSOV, Alexey (Mathematical Finance) *Solvable Markov processes*
LAWI, Stephan (Mathematical Finance) *Exactly solvable stochastic integrals and q-deformed processes*
SAVU, Anamaria (Probability) *Hydrodynamic scaling limit of the continuum solid on solid model*
SHAHBAZI, Zohreh (Differential Geometry) *Differential Geometry of Relative Gerbes*
SONG, Joon-Hyeok (Symplectic Geometry) *Intersection Numbers in q-Hamiltonian Spaces*
TIMORIN, Vladlen (Analytic Geometry) *Rectifiable Pencils of Conics*

2005
DE LOS SANTOS, Alejandro (Mathematical Finance) *Liquidity risk estimation: non-gaussian AR models and quantile expansions*
HAMILTON, Mark (Symplectic Geometry) *Singular Bohr-Sommerfeld Leaves and Geometric Quantization*
NIU, Zhuang (Operator Algebras) *A classification of the tracially approximately sub-homogeneous C*-algebras*
PATANKAR, Vijay (Number Theory) *Splitting of Abelian Varieties*
POLLANEN, Marco (Probability) *Low discrepancy sequences in probability spaces*

2006
CHAN, Jackson (Mathematical Physics) *Methods of variations of potential of quasi-periodic Schroedinger equation*
DEJAK, Steven (Nonlinear PDE) *Long-time dynamics of KdV solitary waves over a variable bottom*
DOUGLAS, Andrew (Representation Theory) *A classification of the finite dimensional indecomposable representations of the Euclidean algebra \( \mathfrak{e}(2) \*)
HO, Toan Minh (Operator Algebras) *On the inductive limits of homogeneous algebras with diagonal morphisms between building blocks*
KNAFO, Emmanuel (Number Theory) *Variance of distribution of almost primes in arithmetic progressions*
ROBERT GONZALEZ, Leonel (Operator Algebras) *Classification of nonsimple approximate interval C*-algebras: the triangular case*

APPENDIX D: THE FIELDS INSTITUTE FOR RESEARCH IN MATHEMATICAL SCIENCES

The Fields Institute for Research in Mathematical Sciences was created in November 1991 with major funding from the Province of Ontario, the Natural Sciences and Engineering Research Council of Canada, and McMaster University, the University of Toronto, and the University of Waterloo. In September 1996 it moved from its temporary location in Waterloo to its permanent site, a new building located at 222 College Street in Toronto, next to the University of Toronto Bookstore. In addition to the three principal sponsoring universities about twenty universities across Canada are affiliated with it.

The mandate of the Fields Institute specifically includes the training of graduate students and this function is given a higher profile than at other similar mathematics research institutes. All major programs run at the institution contain graduate courses which students at any university affiliated with the institute may take for credit and the organizers of major programs are expected to set aside some money to make it possible for graduate students to participate in
APPENDIX E: INSTRUCTIONS FOR COURSE ENROLMENT ON ROSI

INSTRUCTIONS FOR COURSE ENROLMENT
ON THE STUDENT WEB SERVICE (SWS)
2006 - 2007

Graduate students are able to access the student web service to change personal information (addresses and telephone numbers), view their academic record and current courses and to enrol in, request or drop courses.

General Information

Student Responsibility

While academic advisors, faculty and staff are available to assist and advise, it is ultimately the student's responsibility to keep his/her personal and academic information up to date at all times and to follow all University, SGS, departmental and program regulations, requirements and deadlines. The student web service makes it easier for students to check and correct this information. If questions arise about requirements, policies and procedures, students are responsible for seeking answers for these questions from staff and advisors.

Note: the department and other university offices may send important information to you by email. Please make sure that your email address, your mailing/permanent address and telephone numbers are up to date at all times. Under University policy students are required to maintain a University based email account (e.g., UTOR, ECF, CHASS, OISE), record that in ROSI and regularly check for messages. That account may be forwarded to another personal account but it is the University account to which the University will send official correspondence.

Declaration

Use of the SWS to enrol in courses means that you agree to abide by all of the academic and non-academic rules and regulations of the University, the School of Graduate Studies and department in which you are registered and assume the obligation to pay academic and incidental fees according to the policies and requirements of the University of Toronto. You normally use the SWS to add or cancel courses. If, for extraordinary reasons, you are unable to use the SWS contact your department office as soon as possible.

Users of the Student Web Service are expected to be responsible when using the SWS and should not attempt to flood the system with requests, or to automate the process of course enrolment. Such activity may clog the system so that other students may be denied access or experience degraded performance. Any student(s) attempting such activity may be denied access to the SWS until after the relevant registration period.

Personal Identification Number

Each time a student accesses ROSI via the web a personal identification is required in addition to a student number. The first time the system is accessed this will be derived from the student's date of birth (format YYYMMDD). However, at that point the student will be required to change the PIN. Subsequent access to the system will require this new number which should be known only to the student. The PIN and student number together constitute an "electronic signature". Never give your PIN or student number to someone else.

Forgotten PIN numbers can be reset by the graduate office. To avoid having to contact the office in person or having to wait for office hours, students can enter answers to a set of questions on the SWS. When they first access the ROSI, the SWS will prompt students to choose three questions from a list. If at a later date the student forgets
the PIN, the PIN can be reset online if two of the three questions are answered correctly.

**Services Available**

- Change PIN number
- View/Change address, telephone number, email
- View final grades
- View academic history
- Add/request/drop courses
- View current courses or course request status
- View student account information
- Order transcripts
- Order graduation tickets
- View transaction log

**Updating Personal Information**

Students may view or update their address, telephone number or email address through the Student Web Service. When entering new information, the "add" option should be used. "Change" should only be used to correct information in an otherwise correct record (e.g. typos).

**Requesting Courses**

Students may begin requesting courses in August. *All course requests must be approved by the graduate coordinator. Students must request their courses by no later than September 29. Courses will be approved or refused before the last date to add courses.*

The web service requires full information about a course when a request is being made. Please consult the attached lists from your department. Be sure to enter:

- **Course number:** e.g. HIS2651Y
- **Section Code:** usually F, S or Y. This indicates whether the course is offered in the fall session (F), the winter session or second term (S) or over both (Y).
- **Teaching Method:** all graduate courses have a teaching method of LEC (lecture).
- **Teaching Section:** the number of the class. Most graduate courses only have one teaching session (0101). Although there may be only one teaching section the information must still be entered on the system.

NB. Some courses may require instructor's approval *in addition to that given by the coordinator/academic advisor.*

**Courses from outside the department**

Not all graduate departments allow students to enrol in courses via the web. Before attempting to add a course outside your department check with your department and the host department about procedures.

**Checking course status**

Students are responsible for knowing the status of their course requests at all times. This information can be obtained via the web service. The following are the possible statuses:

- **REQ:** Course requested. Must be resolved/approved by the last date to add a course.
- **INT:** Course requested pending instructor approval in addition to coordinator's/advisor's approval.
- **APP:** Request approved. Student is enrolled in course.
- **REF:** Request denied. Student is not enrolled and may not make another request for this course via the web during this session.
- **CAN:** Course cancelled (student withdrew from course before deadline)
Cancelling or withdrawing from courses

Students may cancel or withdraw from individual courses using the web service up to certain deadline dates. Before doing this however, students are advised to consult with their advisor or departmental office.

Deadline dates:

**August 1**  
First date students may request courses for the September 2006 and January 2007 sessions

**August 31**  
Payment or deferral date. Fees should be paid at a chartered bank by this date to allow for funds transfer in time for the September 16 registration deadline. Students not registered by the deadline will have their eligibility and courses cancelled and will not be permitted further access to enrol by the SWS.

**September 29:**  
Math department deadline to request fall and full year courses (F, Y sections) "online" for approval by department. Requires submission of departmental enrolment form.

**October 6:**  
Absolute deadline to add fall and full year courses. Students will not be considered enrolled unless they have a course status of "APP".

**November 3:**  
Last date to ‘cancel’ (i.e. withdraw) from a fall (F) course. Requires submission of departmental program change form.

**January 15:**  
Math department deadline to request winter session/second term (S) on ROSI. Requires submission of departmental program change form

**January 19:**  
Last date for students to request winter session/second term (S) courses. Courses requiring approval must be cleared with the department before this date.

**March 2:**  
Last date to ‘cancel’ (i.e. withdraw) from a full year (Y) or winter session/second term course. Requires submission of departmental program change form.

Final Results

Final grades in courses can be accessed through “Transcripts and Academic History”. Grades can be viewed after the following dates. If a grade is not available, contact your instructor or the graduate unit offering the course.

<table>
<thead>
<tr>
<th>Session</th>
<th>Date</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006 Summer Session (first term)</td>
<td>August 2</td>
</tr>
<tr>
<td>(full summer and second term)</td>
<td>September 20</td>
</tr>
<tr>
<td>2006 Fall Session</td>
<td>January 24</td>
</tr>
<tr>
<td>2007 Winter Session (and Fall/Winter courses)</td>
<td>May 23</td>
</tr>
</tbody>
</table>

System Availability

The student web service is normally available at the following times:

- **Monday** 6:00 to 23:45
- **Tuesday to Thursday** 0:15 to 23:45
### Friday
0:15 to 18:00

### Saturday
Midnight to midnight

### Sunday
Midnight to 23:45

Occasionally hours must be reduced for system maintenance. Please check the Student Web Service for details.

**URL**

The Student Web Service (a.k.a. ROSI's Page) can be accessed at [www.rosi.utoronto.ca](http://www.rosi.utoronto.ca). Instructions are located there. Please remember to log out after each use.

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**APPENDIX F: SGS ACADEMIC CALENDAR 2006-2007**

<table>
<thead>
<tr>
<th>Year</th>
<th>Month</th>
<th>Day</th>
<th>Event Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2006</td>
<td>August</td>
<td>7</td>
<td>Civic Holiday</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>14</td>
<td>Registration for September session begins</td>
</tr>
<tr>
<td></td>
<td>August</td>
<td>31</td>
<td>Last date for payment of tuition fees to meet registration deadline</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>4</td>
<td>Labour Day</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>11</td>
<td>Graduate courses and seminars begin in the week of September 11&lt;sup&gt;(1)&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>15</td>
<td>Final date to submit Ph.D. theses to SGS to avoid fee charges for 2006-07</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>15</td>
<td>Registration for September session ends; after this date, a late registration fee will be assessed</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>15</td>
<td>Coursework must be completed and grades submitted for summer session courses and extended courses&lt;sup&gt;(2)&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>September</td>
<td>20</td>
<td>Summer Session grades available for viewing by students on the SWS</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>6</td>
<td>Final date for receipt of degree recommendations and submission of any required theses for master's degrees for Fall Convocation&lt;sup&gt;(3)&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>6</td>
<td>Final date to submit final Ph.D. thesis for Fall Convocation</td>
</tr>
<tr>
<td></td>
<td>October</td>
<td>9</td>
<td>Thanksgiving Day</td>
</tr>
<tr>
<td></td>
<td>November</td>
<td>3</td>
<td>Final date to drop September session full or half courses without academic penalty</td>
</tr>
<tr>
<td></td>
<td>November</td>
<td>TBA</td>
<td>Fall Convocation</td>
</tr>
<tr>
<td></td>
<td>November</td>
<td>TBA</td>
<td>Fall Convocation</td>
</tr>
<tr>
<td></td>
<td>December</td>
<td></td>
<td>For last day of classes before Winter break, consult graduate units</td>
</tr>
</tbody>
</table>

**2007**
<table>
<thead>
<tr>
<th>Date</th>
<th>Event</th>
</tr>
</thead>
<tbody>
<tr>
<td>M Jan 8</td>
<td>Graduate courses and seminars begin in the week of January 8th (1)</td>
</tr>
<tr>
<td>F Jan 12</td>
<td>Final date for registration of students beginning program in January session; after this date, a late registration fee will be assessed</td>
</tr>
<tr>
<td>M Jan 15</td>
<td>Final date to submit Ph.D. theses without fee payment for January session</td>
</tr>
<tr>
<td>F Jan 19</td>
<td>Coursework must be completed and grades submitted for September session courses (2)</td>
</tr>
<tr>
<td>F Jan 19</td>
<td>Final date to add January session courses (4)</td>
</tr>
<tr>
<td>W Jan 24</td>
<td>September Session grades available for viewing by students on the Student Web Service</td>
</tr>
<tr>
<td>F Jan 26</td>
<td>Final date for receipt of degree recommendations and submission of any required theses for March or June graduation for master's students without fees being charged for the January session (3)</td>
</tr>
<tr>
<td>F Jan 26</td>
<td>Final date for all students to request that their degrees be conferred in absentia in March</td>
</tr>
<tr>
<td>F Jan 26</td>
<td>September dual registrants must be recommended for the master's degree by this date to maintain their Ph.D. registration (3)</td>
</tr>
<tr>
<td>F Mar 2</td>
<td>Final date to drop full-year or January session courses without academic penalty (4)</td>
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<tr>
<td>March TBA</td>
<td>March Graduation in absentia</td>
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<td>F Apr 13</td>
<td>Good Friday</td>
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<tr>
<td>F Apr 27</td>
<td>For students obtaining degrees at June Convocation, course work must be completed and grades submitted for full-year and January session courses</td>
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<tr>
<td>F Apr 27</td>
<td>Final date for receipt of degree recommendations and submission of any required theses for master’s degrees for June Convocation (3)</td>
</tr>
<tr>
<td>F Apr 27</td>
<td>Final date for submission of final Ph.D. thesis for students whose degrees are to be conferred at the June Convocation</td>
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<tr>
<td>F Apr 27</td>
<td>Final date for degree recommendations of January dual registrants for the master's degree to maintain their Ph.D. registration (3)</td>
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<tr>
<td>May</td>
<td>For first day of summer classes, consult graduate unit concerned.</td>
</tr>
<tr>
<td>F May 4</td>
<td>Final date for registration for May session</td>
</tr>
<tr>
<td>F May 18</td>
<td>Final date to enrol in May-June or May-August session courses</td>
</tr>
<tr>
<td>F May 18</td>
<td>Course work must be completed and grades submitted for full-year and January session courses (except for extended courses) (2)</td>
</tr>
<tr>
<td>M May 21</td>
<td>Victoria Day</td>
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<td>Day</td>
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<td>Aug</td>
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*(1) The precise dates of commencement of courses are determined by the graduate units; students are urged to contact the relevant graduate units for information. The University policy states that the first day of classes in the September session in all teaching divisions should not be scheduled on the first and second days of Rosh Hashanah (from 1 1/2 hours before sunset on Friday, September 22nd to about 1 1/2 hours after sunset on Sunday, September 24th) or on Yom Kippur (from about 1 1/2 hours before sunset on Sunday, October 1st to about 1 1/2 hours after sunset on Monday, October 2nd).

*(2) Graduate units may establish earlier deadlines for completion of course work and may prescribe penalties for late completion of work and for failure to complete work, provided that these penalties are announced at the time the instructor makes known to the class the methods by which student performance shall be evaluated.

*(3) For final dates for completing degree requirements, students should consult their own departments.

*(4) Graduate units may establish earlier deadlines to add/drop courses. Course changes for part-time special students require an earlier deadline.