1- Answer by TRUE or FALSE the following:

1. If \( f \) is an expanding map, then it implies that it is chaotic.
   True  False

2. If \( f \) is expanding then it is expansive.
   True  False

3. If \( f \) is expanding and the derivative is continuous then it is expansive.
   True  False

4. Let \( f \) be a continuous function. If the periodic points are dense then the function is expansive.
   True  False

5. If \( f \) is continuous and has a periodic point of period three then it is not one to one.
   True  False

6. If \( f \) has a periodic point of period three then is not one to one.
   True  False

7. Let \( f : I \to I \) be an injective (one to one) continuous increasing function then it has a fixed point.
   True  False

2- Prove the following:

1. If \( f \) has an attracting periodic point then the periodic points are not dense.

2. If \( f \) has an attracting periodic point then \( f \) is not expansive.

3. Then map \( f(x) = 4x(1 - x) \) is not expansive.

4. Let \( f : \mathbb{R} \to \mathbb{R} \) be an injective (one to one) continuous function then it is not chaotic. Does it have fixed points? If it is decreasing?

5. Let \( f \) be a derivable function. If \( p \) is a fixed point of \( f \) such that it attracts from one side and repells from the other side. Then \( p \) is neutral fixed point \((|f'(p)| = 1)).

3- Let \( F(x) = 4x(1 - x) \).

1. Sketch the graph and find the fixed points. Are attracting or repelling? Justify.

2. Does it have periodic point of arbitrarily large period? Justify.

3. Is \( F \) restricted to the interval \([0, 1]\) expansive? Justify.
4. Find the set \( \{ x : F^n(x) \rightarrow +\infty \} \).

5. Find the set \( \{ x : F^n(x) \rightarrow -\infty \} \).

3- Let \( F(x) = 10x(1 - x) \).

1. Sketch the graph and find the fixed points. Are attracting or repelling? Justify.

2. Does it have periodic point of arbitrarily large period? Justify.

3. Find the set \( \{ x : F^n(x) \rightarrow +\infty \} \).

4. Find the set \( \{ x : F^n(x) \rightarrow -\infty \} \).

4- Let \( L : [0, 3] \rightarrow [0, 3] \) be the function.

\[
L(x) = \begin{cases} 
2x + x^3 & 0 \leq x \leq 1 \\
\frac{3}{2}x - \frac{3}{2} & 1 < x \leq 3 
\end{cases}
\]

1. Try to sketch the graph and find the fixed points. Are attracting or repelling? Justify.

2. How does the graph of \( L^n \) look for any positive integer \( n \)?

3. How many periodic points of period \( n \) does \( L \) have?

4. How many periodic points of period 30 does \( L \) have? Justify.

5. Are the periodic point dense? Justify.


7. Explain how it is constructed a symbolic dynamic induced by \( L \). Show that \( L \) is conjugate to the symbolic dynamics.