Neutral fixed points and bifurcations.

1- Suppose that $F$ has a neutral fixed point at $x_0$ with $F'(x_0) = 1$.
   1. Suppose also that $F''(x_0) > 0$. What can you say about $x_0$: is it attracting, repelling, one-side attracting, one-side repelling?
   2. Idem as in 5 but assuming now that $F''(x_0) < 0$.

2- Suppose that $F$ has a neutral fixed point at $x_0$ with $F'(x_0) = 1$ and $F''(x_0) = 0$.
   1. Suppose also that $F'''(x_0) > 0$. What can you say about $x_0$: is it attracting, repelling, weakly attracting, repelling?
   2. Idem as before but assuming now that $F'''(x_0) < 0$.

3- Each of the following function has a neutral fixed point. Find this point and determine the type of it.
   1. $F(x) = x + x^2$
   2. $F(x) = x - x^2$
   3. $F(x) = -x - x^2$
   4. $F(x) = -x + x^2$
   5. $F(x) = \frac{1}{x}$
   6. $F(x) = -\frac{1}{2}x^3 - \frac{3}{2}x^2 + 1$
   7. $F(x) = \exp(x - 1)$ (fixed point is $X_0 = 1$).
   8. $F(x) = \sin(x)$
   9. $F(x) = \tan(x)$
   10. $F(x) = x + x^3$
    11. $F(x) = x - x^3$
    12. $F(x) = -x + x^3$
13. $F(x) = -x - x^3$
14. $F(x) = \log(|x - 1|)$

4- Each of the following functions undergoes a bifurcation of fixed point at the given parameter value. In each case, identify the type of the bifurcation. In each case, identify the phase portrait of the bifurcation.

1. $F_\lambda(x) = x + x^2 + \lambda, \lambda = 0$
2. $F_\lambda(x) = x + x^2 + \lambda, \lambda = 1$
3. $F_\mu(x) = \mu x + x^3, \mu = 1$
4. $F_\mu(x) = \mu x + x^3, \mu = 1$
5. $F_\mu(x) = \mu \sin(x), \mu = 1$
6. $F_\mu(x) = \mu \sin(x), \mu = 1$
7. $F_c(x) = x^3 + c, c = \frac{2}{3\sqrt{3}}$
8. $F_\lambda(x) = \lambda(\exp(x)1), \lambda = 1$
9. $F_\lambda(x) = \lambda(\exp(x)1), \lambda = 1$
10. $F_c(x) = cx^2 + x, c = 0$
11. $F_c(x) = x^3 + cx^2 + x, c = 0$

5- Consider the family $f_\mu(x) = x^3 + \frac{9}{2}x^2 + (5 + \mu)x + \frac{1}{2}$ with $\mu$ close to 0.

1. Sketch the graph of $f$. Try to localize the fixed point of $f_\mu$ for $\mu = 0$.
2. Show that the family has only one neutral fixed point for $\mu = 0$. Is it attracting or repelling? Justify.
3. Study the phase portrait of the bifurcation of the neutral fixed point.

6- Let $f_\lambda$ be the family $f_\lambda(x) = \lambda x - x^3$. Shows that there is a periodic point of period two for $\lambda > -1$. Is it repelling or attracting? Justify.

7- Considering the quadratic family $Q_c(x) = x^2 + c$

1. Prove that for $\frac{3}{4} < c < \frac{3}{4}$ there is an attracting periodic point of period two.
2. Prove that for $c = \frac{3}{4}$ there is a neutral periodic point of period two.
3. Prove that for $c = \frac{3}{4}$ there is a repelling periodic point of period two.

7- Consider the quadratic family $F_\mu(x) = x^2 - \mu$ with $\mu \in [1,9]$. 

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1. For each $\mu$ sketch the graph and find the fixed points. Are attracting or repelling? Justify.

2. For which values of $\mu$ there exists an invariant interval?

3. For which $\mu$ there are periodic point of arbitrarily large period? Justify.

4. For each $\mu$ find the set $\{x : F^\mu_\mu(x) \to +\infty\}$.

5. For each $\mu$ find the set $\{x : F^\mu_\mu(x) \to -\infty\}$.

6. For which parameter of $\mu$ there is a bifurcation?