Homework Assignment 8

Assigned Tuesday October 29; due Friday November 8, 2PM at SS 1071

web version: http://www.math.toronto.edu/~drorbn/classes/0203/157AnalysisI/HW08/HW08.html

Required reading
All of Spivak Chapter 9.

To be handed in
From Spivak Chapter 9: 1, 9, 15, 23.

Recommended for extra practice
From Spivak Chapter 9: 8, 11, 21, 28.

Also, let \( p(x) \) be the polynomial \( x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0 \). Now that we know that for \(|x| > 2n \max(|a_{n-1}|, \ldots, |a_1|, |a_0|, 1) \) we have that
\[
\frac{1}{2}|x^n| > |a_{n-1}x^{n-1} + \cdots + a_1x + a_0|,
\]
complete the proof of the following

Theorem.

- If \( n \) is odd then the equation \( p(x) = c \) has a root for any value of \( c \).

- If \( n \) is even then there is some constant \( c_0 \) so that the equation \( p(x) = c \) has no roots for \( c < c_0 \), has at least one root for \( c = c_0 \) and at least two roots for \( c > c_0 \).

Just for fun

Write a computer program that will allow you to draw the graph of the function
\[
f(x) = \sum_{n=0}^{\infty} \frac{1}{2^n} \sin 3^n x,
\]
and will allow you to zoom on that graph through various small “windows”. Use your program to convince yourself that \( f \) is everywhere continuous but nowhere differentiable. The best plots will be posted on this web site! (Send pictures along with window coordinates by email to drorbn@math.toronto.edu).