I Understand Khovanov Homology

The Jones polynomial:

Definition. $\hat{J} : \bigotimes \mapsto q^2 \bigotimes - q^{-1} \bigotimes, \quad J : \bigotimes \mapsto -q^{-2} \bigotimes + q^{-1} \bigotimes$.

Example / Khovanov Homology:

\[
(q + q^{-1})^2 - 3q(q + q^{-1}) + 3q^2(q + q^{-1})^2 - q^3(q + q^{-1})^3
\]

Who cares?

4D People

4D People

M. Jacobsson

J. Rasmussen

Drinfel'd

Everything in knot theory comes from quantum groups

Conjecture: (I. Frenkel, though he may disown this version)
1. Every object in mathematics is the Euler characteristic of a complex.
2. Every operation in mathematics lifts to an operation between complexes.
3. Every identity in mathematics remains true up to homotopy.

Witten:
Everything in knot theory comes from physics

HOMFLY–PT:

\[
aH(\bigotimes) - a^{-1}H(\bigotimes) = zH(\bigotimes); \quad H(\bigotimes) = 1
\]

\[
q^n P_n(\bigotimes) - q^{-n} P_n(\bigotimes) = (q - q^{-1}) P_n(\bigotimes);
\]

\[
P_n(\bigotimes) = \frac{q^n - q^{-n}}{q - q^{-1}}
\]

Invariance under R2:

\[
J : \bigotimes \mapsto -q^{-1} \bigotimes + \bigotimes - q^2 \bigotimes - q^{-2} \bigotimes = -q^{-1} \bigotimes + (q + q^{-1}) \bigotimes - q^2 \bigotimes
\]