MAT 495 HOMEWORK 5: NUMBER THEORY

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Write up and submit any 4 problems \textit{with proofs} by Friday, November 20th. Points will be deducted for missing cases, gaps in the proof, algebra mistakes, and errors in reasoning, so please write very carefully!

If you can’t solve 4 problems, you can submit partial work, but please do not submit work on more than 4 problems.

1. Problems

(1) Find all positive integers $n$ such that $n^2 + 3n + 2$ is a power of 2.
(A power of 2 is a number of the form $2^n$, where $n$ is a non-negative integer. $1, 2, 4, \ldots$)

(2) Find all pairs of positive integers $x, y$ such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{1841}$.

(3) Find all primes numbers $p$ such that $p$ can be written as a sum of two prime numbers, and $p$ can be written as a difference of two prime numbers.

(4) Let $m$ be the number 11111...111 with 2014 digits, all of them 1. What is the 1008'th digit of $\sqrt{m}$ after the decimal point?

(5) Suppose that $n$ is an integer greater than 1 such that $n + 1$ is not prime. Prove that $n + 1$ and $n! + 1$ are relatively prime.

(6) Prove that the product of four consecutive natural numbers is never a perfect square. \textit{Hint: Do a few small cases! What do you notice?}

(7) Find all pairs of primes $p, q$ such that $2^{pq} + 1$ is divisible by $pq$.

(8) Let $a, b, c, d$ be positive integers with $ab = cd$. Prove that $a + b + c + d$ is not prime.

(9) (Hard!) Prove that if $1 + 2^n + 4^n$ is a prime number, then $n$ is a power of 3.