(1) Let T be a (2,1) tensor on a manifold M^n such that $T(X,Y) = -T(Y,X)$ for any $p \in M$ and any vectors $X, Y \in T_pM$. Prove that there exists a connection ∇ on M such that its torsion is equal by T.

(2) Finish the proof of the theorem from class.

Let M^n be a manifold and ∇ be a connection on M. Let $\gamma\colon \mathbb{R} \to M$ be a smooth curve in M and let $\mathfrak{X}_\gamma$ be the space of smooth vector fields along γ

Then exists a unique operation $\frac{D}{dt}\colon \mathfrak{X}_\gamma \to \mathfrak{X}_\gamma$ satisfying the following conditions

(a) $\frac{D}{dt}(Y_1 + Y_2) = \frac{D}{dt}Y_1 + \frac{D}{dt}Y_2$

(b) $\frac{D}{dt}(fY) = f'(t)Y + f(t)\frac{D}{dt}Y$ for any $f \in C^\infty(\mathbb{R})$.

(c) If $Y = \tilde{Y} \circ \gamma$ for some vector filed $\tilde{Y}$ on M then $\frac{D}{dt}Y(t) = \nabla_{\gamma'(t)}\tilde{Y}(\gamma(t))$

(3) Prove that parallel transport along a curve γ is independent of the parametrization of γ.

In other words, let $\gamma\colon [0,1] \to M$ be a smooth curve in a manifold M. Let $p = \gamma(0), q = \gamma(1)$ and let $v \in T_pM$. Let $w \in T_qM$ be the result of parallel transport of v along γ.

Now let $\alpha\colon [0,1] \to [0,1]$ be a diffeomorphism fixing the end points. Let $\tilde{\gamma} = \gamma \circ \alpha$. Let $\tilde{w} \in T_qM$ be the result of the parallel transport of v along $\tilde{\gamma}$.

prove that $w = \tilde{w}$.