(1) Let σ be a singular 3-cube in R given by $\sigma(x_1, x_2, x_3) = x_1$. where $(x_1, x_2, x_3) \in [0,1]^3$.

Find $\partial\sigma$.

(2) Let $\sigma\colon [0,1]^2 \to R^3$ be given by $\sigma(x, y) = (xy, 2x + y, y^2)$. Let ω be a 2-form on R^3 given by $x_1 dx_2 \wedge dx_3 + x_2^2 dx_1 \wedge dx_3$ and let η be a 1 form given by $\eta = y^2 dx + x^2 dy$.

(a) Find $\int_\sigma \omega$.

(b) Find $\int_\sigma d\eta$ and $\int_{\partial\sigma} \eta$ and verify that they are equal.

(3) Let V be a vector space and $W \subset V$ be a subspace. we'll say that a vector v is equivalent to v' if $v + W = v' + W$ as sets. we'll denote this by $v \sim v'$.

(a) Show that if $v \sim v'$ and $v' \sim v''$ then $v \sim v''$.

(b) Show that $v \sim v'$ iff $v \subset v' + W$.

This means that V can be represented as a union of distinct equivalence classes $[v]$ and the equivalence class of v is equal to $v + W$.

For two equivalence classes $[v_1], [v_2]$ define $[v]_1 + [v_2] := [v_1 + v_2]$ and $\lambda[v_1] := [\lambda v_1]$

(a) verify that the above operations are well defined.

(b) verify that the set of equivalence classes with the above operation is a vector space. This space is called the quotient space of V by W and is denoted by V/W.