

Practice Final 1

- What is $\phi(20^{100})$ where ϕ is Euler's ϕ -function?
 - Find an integer x such that $140x \equiv 133 \pmod{301}$.
Hint: $\gcd(140, 301) = 7$.
- Prove, by mathematical induction, that $1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$ for every natural number n .
 - Prove that for p an odd prime (that is, p is a prime that is not equal to 2), $1^p + 2^p + 3^p + \dots + (p-1)^p \equiv 0 \pmod{p}$.
- Prove that for any odd integer a , a and a^{4n+1} have the same last digit for every natural number n .
- Recall that a "perfect square" is a number of the form n^2 where n is a natural number. Show that 9120342526523 is not the sum of two perfect squares. *Hint:* Consider values modulo 4.
- Are there rational numbers a and b such that $\sqrt{3} = a + b\sqrt{2}$? Justify your answer.
 - Prove that $\frac{\sqrt{5}}{\sqrt{2+\sqrt{11}}}$ is irrational.
- What is the cardinality of the set of roots of polynomials with constructible coefficients? Justify your answer.
 - Let \mathbb{N} denote the set of all natural numbers. What is the cardinality of the set of all functions from \mathbb{N} to $\{1, 3, 5\}$? Justify your answer.
Hint: You can use the fact that $|P(\mathbb{N})| = |\mathbb{R}|$.
- Let θ be an angle between 0 and 90 degrees. Suppose that $\cos \theta = \frac{3}{4}$. Prove that $\frac{\theta}{3}$ is not a constructible angle.
- For each of the following numbers, state whether or not it is constructible and justify your answer.
 - $\cos \theta$ where the angle $\frac{\theta}{3}$ is constructible
 - $\sqrt{7 + \sqrt{5}}$
 - $(0.029)^{1/3}$
 - $\tan 22.5^\circ$
- Find all complex solutions of the equation $z^6 + z^3 + 1 = 0$.
- Let $p = 3, q = 11$ and $e = 7$. Let $N = 3 \cdot 11 = 33$. The receiver broadcasts the numbers $N = 33, e = 7$. The sender sends a secret message M to the receiver using RSA encryption. What is sent is the number $R = 6$.
Decode to find the original message M .

11. Construct a polynomial with integer coefficients which has $\sqrt{2} + \sqrt{5}$ as a root.