MAT 246S **Practice Problems**

(1) Compute $\sqrt{3}$ up to 2 decimal points without using a calculator.

(2) Recall that a triple of natural numbers a, b, c is called Pythagorean if $a^2 + b^2 = c^2$.

Find all Pythagorean triples with $b = 60$.

(3) Using Euclidean algorithm prove that if $a|mb$, $(a, b) = 1$ then $a|m$. You are NOT allowed to use the uniqueness of prime factorization theorem.

(4) Consider the binomial expansion $(a + b)^n = C_0^n a^n + C_1^n a^{n-1} b + \ldots + C_n^n b^n$.

Show that $C_0^n + C_1^n + C_2^n + \ldots + C_n^n = 2^n$

(5) Prove that

$$1 + 2q + 3q^2 + \ldots + nq^{n-1} = \frac{1 - (n+1)q^n + nq^{n+1}}{(1-q)^2}$$

(6) Let $(a, p) = 1$ where p is prime. prove that there exists a number b such that $ab \equiv 1 \pmod{p}$. You are NOT allowed to use any theorems when proving this.

(7) Prove that $|\mathbb{R}| \leq |P(\mathbb{N})|$.

(8) Prove that $|\mathbb{R}| \geq |P(\mathbb{N})|$.

Hint: represent a subset A of N a sequence of 1s and 0s such that the n-th element of the sequence is 1 if $n \in A$ and is 0 if $n \notin A$.

(9) Find the formula for the following sum and prove it by induction $1 - q^3 + q^6 - \ldots + (-1)^n q^{3n}$.

(10) Let $a > 0$ be algebraic. Prove that $\sqrt{a}$ is algebraic.

(11) let a be algebraic and let q be rational. Prove that qa is also algebraic.

(12) Prove that if $a > 0$ is constructible then $1/a$ is also constructible.

(13) Show that if $a, b \in \mathbb{N}$ and $a^3|b^3$ then $a|b$.