

You do not need to turn in these problems. They are just practice problems on the material of the last lecture.

- (1) (a) Let $f(x) = a_n x^n + \dots a_1 x + a_0$ be a polynomial with integer coefficients. Suppose $x_0 = \frac{p}{q}$ is a root of $f(x) = 0$ where $p, q \in \mathbb{Z}$ and $(p, q) = 1$. Show that $p|a_0$ and $q|a_n$.
- (b) Show that the equation $3x^5 - 2x^3 - x^2 - 2 = 0$ has no rational solutions.

- (2) Find all constructible roots of the equation

$$x^4 - 2x^3 - 2x^2 + 9x - 10 = 0$$

- (3) Let $0 < \theta < \pi/2$ be the angle with $\cos \theta = \frac{2}{7}$. Show that θ is constructible but $\theta/3$ is not.
- (4) Show that the equation $x^9 - 4x^3 + 1 = 0$ has no constructible roots.