Problem 1. Let \( f : \mathbb{R}^2 \to [0, \infty) \) be the map such that \( f(x, y) = \sqrt{x^2 + y^2} \). Show that \( f \) is a quotient map.

Problem 2. Show that the circle (with the standard topology) arises as a quotient space of the real line \( \mathbb{R} \) (with the standard topology).

Problem 3. Construct an example for a quotient space of \( \mathbb{R} \) (with the standard topology) which is not Hausdorff.

Problem 4. Let \( C = \{(x, y)|x^2 + y^2 = 1\} \) be the circle with the standard topology. Let \( \mathcal{P} \) be the partition on \( C \) whose partition sets are the pairs \( \{(x, y), (-x, -y)\} \). Show that the quotient space according to \( \mathcal{P} \) is homeomorphic to the circle.