List of publications

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In preparation:

[1] Pavel Bleher, Mikhail Lyubich, and Roland K. W. Roeder. *Complex dynamics for the Yang-Lee zeros.*

Please see my research statement, Section 1.1, for a partial description.

[2] Suzanne Lynch Hruska and Roland K. W. Roeder. *Topology of Fatou components for polynomial skew products of* $\mathbb{P}^2$.

Please see my research statement, Section 1.2, for a partial description.

Published (or "to appear"):

[1] Omar Antolín-Camarena, Gregory R. Maloney, and Roland K. W. Roeder. *Computing arithmetic invariants for hyperbolic reflection groups.* Conditionally accepted to appear in a volume celebrating John Hubbard's 60th birthday. See also: arXiv:0708.2109

We describe a collection of computer scripts written in PARI/GP to compute, for reflection groups determined by finite-volume polyhedra in $\mathbb{H}^3$, the commensurability invariants known as the invariant trace field and invariant quaternion algebra. Our scripts also allow one to determine arithmeticity of such groups and the isomorphism class of the invariant quaternion algebra by analyzing its ramification.

We present many computed examples of these invariants. This is enough to show that most of the groups that we consider are pairwise incommensurable. For pairs of groups with identical invariants, not all is lost: when both groups are arithmetic, having identical invariants guarantees commensurability. We discover some "unexpected" commensurable pairs this way. We also present a non-arithmetic pair with identical invariants for which we cannot determine commensurability. Additionally we complete the the classification of arithmetic Löbell polyhedra obtained by Vesnin, showing that the n-th Löbell polyhedron is arithmetic if and only if $n = 5, 6$, or 8.

[2] Roland K. Roeder. *Constructing hyperbolic polyhedra using Newton's Method.* To appear in Experimental Mathematics issue 16(4) 2007; accepted Nov 2006.

We demonstrate how to construct three-dimensional compact hyperbolic polyhedra using Newton's Method. Under the restriction that the dihedral angles are non-obtuse, Andreev's Theorem provides as necessary and sufficient conditions five classes of linear inequalities for the dihedral angles of a compact hyperbolic polyhedron realizing a given combinatorial structure C. Andreev's Theorem also shows that the resulting polyhedron is unique, up to hyperbolic isometry. Our construction uses Newton's method and a homotopy to explicitly follow the existence proof presented by Andreev, providing both a very clear illustration of proof of Andreev's Theorem as well as a convenient way to construct three-dimensional compact hyperbolic polyhedra having non-obtuse dihedral angles.

As an application, we construct compact hyperbolic polyhedra having dihedral angles that are (proper) integer sub-multiples of π, so that the group Γ generated by reflections in the faces is a discrete group of isometries of hyperbolic space. The quotient $\mathbb{H}^3/\Gamma$ is hence a compact hyperbolic 3-orbifold, of which we study the hyperbolic volume and spectrum of closed geodesic lengths using SnapPea. One consequence is a volume estimate for a "hyperelliptic" manifold considered by Mednykh and Vesnin.

[3] Roland K. W. Roeder. *A degenerate newton's map in two complex variables: linking with currents.* J. Geometric Analysis, 17(1):107–146, 2007.

Little is known about the global structure of the basins of attraction of Newton's method in two or more complex variables. We make the first steps by focusing on the specific Newton mapping to solve for the common roots of $P(x, y) = x(1-x)$ and $Q(x, y) = y^2 + Bxy - y$.

There are invariant circles S_0 and S_1 within the lines $x = 0$ and $x = 1$ which are superattracting in the x-direction and hyperbolically repelling within the vertical line. We show that S_0 and S_1 have local super-stable manifolds, which when pulled back under iterates of N form global super-stable spaces W_0 and W_1. By blowing-up the points of indeterminacy p and q of N and all of their inverse images under N we prove that W_0 and W_1 are real-analytic varieties.

We define linking between closed 1-cycles in W_i $(i = 0, 1)$ and an appropriate closed 2 current providing a homomorphism $lk : H_1(W_i, \mathbb{Z}) \to \mathbb{Q}$. If W_i intersects the critical value locus of N, this homomorphism has dense image, proving that $H_1(W_i, \mathbb{Z})$ is infinitely generated. Using the Mayer-Vietoris exact sequence and an algebraic trick, we show that the same is true for the closures of the basins of the roots $\overline{W(r_i)}$.

[4] Roland K. W. Roeder John H. Hubbard and William D. Dunbar. *Andreev's theorem on hyperbolic polyhedra.* Les Annales de l'Institute Fourier, 57(3):825–882, 2007.

In 1970, E. M. Andreev published a classification of all three-dimensional compact hyperbolic polyhedra having non-obtuse dihedral angles. Given a combinatorial description of a polyhedron, C, Andreev's Theorem provides five classes of linear inequalities, depending on C, for the dihedral angles, which are necessary and sufficient conditions for the existence of a hyperbolic polyhedron realizing C with the assigned dihedral angles. Andreev's Theorem also shows that the resulting polyhedron is unique, up to hyperbolic isometry.

Andreev's Theorem is both an interesting statement about the geometry of hyperbolic 3-dimensional space, as well as a fundamental tool used in the proof for Thurston's Hyperbolization Theorem for 3-dimensional Haken manifolds. It is also remarkable to what level the proof of Andreev's Theorem resembles (in a simpler way) the proof of Thurston.

We correct a fundamental error in Andreev's proof of existence and also provide a readable new proof of the other parts of the proof of Andreev's Theorem, because Andreev's paper has the reputation of being "unreadable".

[5] Roland K. W. Roeder *Compact hyperbolic tetrahedra with non-obtuse dihedral angles.* Publicacions Matemàtiques, 50 (1): 211-227, 2006 .

Given a combinatorial description C of a polyhedron having E edges, the space of di-

hedral angles of all compact hyperbolic polyhedra that realize C is generally not a convex subset of $\mathbb{R}^E$. If C has five or more faces, Andreev's Theorem states that the corresponding space of dihedral angles A_C obtained by restricting to *non-obtuse* angles is a convex polytope. In this paper we explain why Andreev did not consider tetrahedra, the only polyhedra having fewer than five faces, by demonstrating that the space of dihedral angles of compact hyperbolic tetrahedra, after restricting to non-obtuse angles, is non-convex. Our proof provides a simple example of the "method of continuity", the technique used in classification theorems on polyhedra by Alexandrow, Andreev, and Rivin-Hodgson.

[6] T.E. Evans, R.K.W. Roeder, J.A. Carter, and B.I. Rapoport. *Homoclinic tangles, bifurcations and edge stochasticity in diverted tokamaks.* Contributions to Plasma Physics, 44 (1-3): 235–240, 2004.

The boundary and pedestal region of a poloidally diverted tokamak is particularly susceptible to the onset of vacuum magnetic field stochasticity due to small non-axisymmetric resonant perturbations. Recent calculations of the separatrix topology in diverted tokamaks, when subjected to small magnetic perturbations, show the existence of complex invariant manifold structures known as homoclinic tangles. These structures appear above a relatively low perturbation threshold that depends on certain equilibrium shape parameters. Homoclinic tangles represent a splitting of the unperturbed separatrix into stable and unstable invariant manifolds associated with each X-point (hyperbolic point). The manifolds that make up homoclinic tangles set the boundaries that prescribe how stochastic field line trajectories are organized i.e., how field lines from the inner domain of the unperturbed separatrix mix and are transported to plasma facing surfaces such as divertor target plates and protruding baffle structures. Thus, the topology of these tangles determines which plasma facing components are most likely to interact with escaping magnetic field lines and the parallel heat and particle flux they carry.

[7] R. K. W. Roeder, B. I. Rapoport, and T. E. Evans. *Explicit calculations of homoclinic tangles in tokamaks.* Phys. Plasmas, 10 (9): 3796–3799, 2003.

Explicit numerical calculations of homoclinic tangles are presented for a physically realistic model of a resonantly perturbed magnetic field in a tokamak. The structure of these tangles is consistent with that expected from the general theory of near-integrable Hamiltonian systems commonly studied with simple algebraic twist map models. In addition, understanding the structure of homoclinic tangles corresponding to the primary separatrix of a poloidally diverted tokamak allows one to make predictions of the locations and structure of magnetic footprints and heat buildup on the tokamak wall. These separatrix tangles undergo an interesting bifurcation sequence as the current through a set of error field correction coils is increased. Since this model of the magnetic field is very realistic, these features are expected to be experimentally verifiable.

[8] Roland K. W. Roeder. *On Poincaré's fourth and fifth examples of limit cycles at infinity.* Rocky Mountain J. Math., 33 (3): 1057–1082, 2003.

Errors are found in example problems from Henri Poincaré's paper, "Mémoire sur les courbes définies par une équation différentielle." Examples Four and Five from Chapter Seven and Examples One, Two and Three from Chapter Nine do not have the limit cycles at infinity predicted by Poincaré. Instead they have fixed points at every point at infinity. In order to understand the errors made by Poincaré, Examples Four and Five are studied at length. Replacement equations for the fourth and fifth Examples are suggested based on the supposition that terms were omitted from Poincaré's equations.

Non-refereed publications:

[**1**] T.E. Evans, R.K.W. Roeder, J.A. Carter et al. *Experimental signatures of homoclinic tangles in poloidally diverted tokamaks.* Journal of Physics: Conference Series, 7: 174-190, 2005.

Small non-axisymmetric perturbations of poloidally diverted tokamaks create edge stochastic magnetic field lines that connect to material surfaces such as those in the divertors. Separatrix structure calculations show that the distribution of stochastic field lines on the vessel walls is closely related to the topology of homoclinic tangles formed in the perturbed system. Since these tangles prescribe how the stochastic fields are organized, they are of significant practical interest in tokamak experiments. Experimental measurements of heat and particle distributions on plasma facing surfaces sometimes show split peak patterns that are consistent with the presence of stochasticity and homoclinic tangles. These split peaks are often observed during locked modes and other types of edge instabilities. They are also observed when perturbation fields from magnetohydrodynamic control coils are pulsed during a plasma discharge. Numerical modeling of the perturbation field from these control coils shows that the homoclinic tangle produced by a coil pulse is not always large enough to produce the splitting patterns observed. Nevertheless, there is a clear correlation between the coil pulses and the appearance of the split profiles. These results suggest the presence of the plasma amplification mechanism that enhances the size of the non-resonant homoclinic tangles.

[**2**] Mahaffy, J., Polk, S. W., Roeder, R. K. W. *An age-structured model for erythropoiesis following a phlebotomy.* Centre de Recherch Mathématiques: CRM-2598, 1999.

This study examines an age-structured model for erythropoiesis following a phlebotomy on normal human subjects. The model extends previous work to include the variable velocity of maturation and the effects of plasma on hematocrit. Experimental data on phlebotomized subjects are used to fit the parameters in the model, and various numerical simulations are performed. The numerical studies are compared to previous models for humans following a blood donation and rabbits with an induced autoimmune hemolytic anemia. The variable aging of precursors to erythrocytes results in increased stability of the model, especially for the anemic rabbit simulations. The relative importance of the various controls in the model and their physiological significance are discussed. Experimental hematocrit readings on two of the authors suggest more complicated controls.

Contributions to textbooks:

[1] Chapter 14, titled: "Andreev's Theorem" in:
Teichmüller Theory and Applications to Geometry, Topology, and Dynamics. Volume II: Four Theorems by William Thurston by John H. Hubbard
with contributions by Adrien Douady, William Dunbar, and **Roland Roeder**, as well as Sylvain Bonnot, David Brown, Allen Hatcher, Chris Hruska, and Sudeb Mitra.
In preparation to be published by Matrix Editions, see
`matrixeditions.com/TeichmullerVol2.html`.

[2] *Teichmüller Theory and Applications to Geometry, Topology, and Dynamics. Volume I: Teichmüller Theory* John H. Hubbard
with contributions by Adrien Douady, William Dunbar, and **Roland Roeder**, as well as Sylvain Bonnot, David Brown, Allen Hatcher, Chris Hruska, and Sudeb Mitra.
Published by Matrix Editions (2006), see
`matrixeditions.com/TeichmullerVol1.html`.