

# Kasra Rafi

## Curriculum Vitae

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### Academic Positions

- 2019–present **Associate Chair (Graduate)**, *University of Toronto.*
- 2018–present **Professor**, *University of Toronto.*
  - 2015–2018 **Associate Professor**, *University of Toronto.*
  - 2012–2015 **Assistant Professor**, *University of Toronto.*
- 2012–present **Member of School of Graduate Studies**, *University of Toronto.*
  - 2008–2012 **Assistant Professor**, *University of Oklahoma.*
  - 2007–2008 **L.E. Dickson Instructor**, *University of Chicago.*
  - 2004–2007 **Postdoctoral Fellow**, *University of Connecticut.*
  - 2001–2004 **Ky Fan Visiting Assistant Professor**, *University of California Santa Barbara.*

### Visiting Positions

- Fall 2016 Simons Visiting Professor at Mathematical Sciences Research Institute
- May-Aug 2010 Hausdorff Research Institute for Mathematics
- Jan-Jul 2009 Clay Research Scholar, University of Chicago
- Fall 2007 Mathematical Sciences Research Institute
- Jul-Aug 2003 Isaac Newton Institute for Mathematical Sciences, University of Cambridge, UK

### Education

- 2001 **Ph.D., Mathematics**, *Stony Brook University, Stony Brook, NY.*
- 1995 **B.Sc., Mathematics**, *Sharif Institute of Technology, Tehran, Iran.*

### Fields of Interest

Geometry and dynamical properties of Teichmüller space  
Geometric group theory, in particular, mapping class group and  $\text{Out}(\mathbb{F}_n)$   
Low dimensional Geometry and Topology

### Grants and Recognitions

- 2020 Fields Institute Fellow
- 2018–2023 NSERC Discovery grant, RGPIN 06486
- Fall 2016 Simons Visiting Professor at MSRI

- 2013–2018 NSERC Discovery grant, RGPIN 435885  
2010–2013 NSF Research Grant PI, DMS-1007811  
Jan-Aug 2009 Clay Research Scholar, University of Chicago  
2001 James Simons Prize for Excellence in Research, Stony Brook University  
1992 Silver Medal, International Mathematics Olympiad, Moscow

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## Research papers

### Preprints

- 2020 **Hyperbolic actions of big mapping class groups**,  
*with Camillie Horbez and Yulan Qing.*
- 2019 **Large Scale Geometry of Big Mapping Class Groups**,  
*with Kathryn Mann.*
- 2019 **Sub-linearly Morse Boundary I: CAT(0) Spaces**,  
*with Yulan Qing and Giulio Tiozzo.*
- 2018 **The shape of a generic translation surface in the minimal stratum**,  
*with Howard Masur and Anja Randecker.*
- 2018 **Geodesics in the Mapping Class Group**,  
*with Yvon Verberne.*
- 2018 **Quasi-geodesics in  $\text{Out}(\mathbb{F}_n)$  and their shadows in sub-factors**,  
*with Yulan Qing.*
- 2017 **Convexity of balls in the outer space**,  
*with Yulan Qing.*

### To Appear

- 2019 **Centralizers in Mapping Class Group and decidability of Thurston Equivalence**,  
*with Nikita Selinger and Michael Yampolsky,*  
to appear in Arnold Mathematical Journal.
- 2018 **Renormalizing an infinite rational IET**,  
*with W. Patrick Hooper and Anja Randecker,*  
to appear in Discrete and Continuous Dynamical Systems - Series A, vol. 40, no. 9; #1.
- 2018 **Local maxima of the systole function**,  
*with Maxime Fortier Bourque,*  
to appear in Journal of the European Mathematical Society.

### Published

- 2020 **Coarse and fine geometry of the Thurston metric**,  
*with David Dumas and Anna Lenzhen and Jing Tao,*  
Forum of Mathematics, Sigma (2020), Vol. 8, e28, 58 pages.
- 2020 **Isomorphisms between big mapping class groups**,  
*with Juliette Bavard and Spencer Dowdall,*  
International Mathematics Research Notices (2020), no. 10, 3084–3099.

- 2020 **Limit sets of Weil-Petersson geodesics with non-minimal ending laminations**,  
*with Jeffrey Brock, Chris Leininger and Babak Modami*,  
*Journal of Topology and Analysis* **12** (2020), no. 1, 1–28.
- 2020 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation II**, *with Jeffrey Brock, Chris Leininger and Babak Modami*,  
*Journal für reine und angewandte Mathematik*, **758** (2020), 1–66.
- 2019 **Limit sets of Weil-Petersson geodesics**,  
*with Jeffrey Brock, Chris Leininger and Babak Modami*,  
*International Mathematics Research Notices* (2019), no. 24, 7604–7658.
- 2019 **Geodesic Currents and Counting Problems**,  
*with Juan Souto*,  
*Geometric and Functional Analysis* **29** (2019), no. 3, 871–889.
- 2019 **Counting closed geodesics in strata**,  
*with Alex Eskin and Maryam Mirzakhani*,  
*Inventiones Mathematicae* **215** (2019), no. 2, 535–607.
- 2018 **Teichmüller geodesics with  $d$ -dimensional limit sets**,  
*with Anna Lenzhen and Babak Modami*,  
*Journal of Modern Dynamics* **12** (2018), 261–283.
- 2018 **Rigidity of Teichmüller space**,  
*with Alex Eskin and Howard Masur*,  
*Geometry & Topology* **22** (2018), no. 7, 4259–4306.
- 2018 **Non-convex balls in the Teichmüller metric**,  
*with Maxime Fortier Bourque*,  
*Journal of Differential Geometry* **110** (2018), no. 3, 379–412.
- 2018 **Algebraic ending laminations and quasi-convexity**,  
*with Mahan Mj*,  
*Algebraic & Geometric Topology* **18** (2018), no. 4, 1883–1916.
- 2018 **Limit sets of Teichmüller geodesics with minimal non-uniquely ergodic vertical foliation**, *with Chris Leininger and Anna Lenzhen*,  
*Journal für die reine und angewandte Mathematik* **737** (2018), 1–32.
- 2017 **Uniform fellow traveling between surgery paths in the sphere graph**,  
*with Matt Clay and Yulan Qing*,  
*Algebraic & Geometric Topology* **17**, (2017), no. 6, 3751–3778.
- 2017 **Large scale rank of Teichmüller space**,  
*with Alex Eskin and Howard Masur*,  
*Duke Mathematical Journal* **166** (2017), no. 8, 1517–1572.
- 2016 **Uniform growth rate**,  
*with Jing Tao*,  
*Proceedings of the American Mathematical Society* **144** (2016), no. 4, 1415–1427.
- 2015 **The shadow of a Thurston geodesic to the curve graph**,  
*with Anna Lenzhen and Jing Tao*,  
*Journal of Topology* **8** (2015), no. 4, 1085–1118.

- 2015 **Digraphs and cycle polynomials for free-by-cyclic groups**,  
*with Yael Algom-Kfir and Eriko Hironaka*,  
*Geometry & Topology* **19** (2015), no 2, 1111–1154.
- 2015 **Mapping tori of small dilatation expanding train-track maps**,  
*with Yael Algom Kfir*,  
*Topology and its Applications* **180** (2015), 44–63.
- 2014 **Uniform hyperbolicity of the curve graph via surgery sequences**,  
*with Matt Clay and Saul Schleimer*,  
*Algebraic & Geometric Topology* **14** (2014), no. 6, 3325–3344.
- 2014 **Hyperbolicity in Teichmüller space**,  
*Geometry & Topology* **18** (2014), no 5, 3025–3053.
- 2014 **On hyperbolicity of free splitting and free factor complexes**,  
*with Ilya Kapovich*,  
*Groups, Geometry, and Dynamics* **8** (2014), no. 2, 391–414.
- 2013 **Diameter of the thick part of moduli space**,  
*with Jing Tao*,  
*Duke Mathematical Journal* **162** (2013), no. 10, 1833–1876.
- 2012 **Bounded combinatorics and the Lipschitz metric on Teichmüller space**,  
*with Anna Lenzhen and Jing Tao*,  
*Geometriae Dedicata* **159** (2012), 353–371.
- 2012 **Grafting rays fellow travel Teichmüller geodesics**,  
*with Young-Eun Choi and David Dumas*,  
*International Mathematics Research Notices* (2012), no. 11, 2445–2492.
- 2011 **Length of a curve is quasi-convex along a Teichmüller geodesic**,  
*with Anna Lenzhen*,  
*Journal of Differential Geometry* **88** (2011), no. 2, 267–295.
- 2011 **Curve complexes are rigid**,  
*with Saul Schleimer*,  
*Duke Mathematical Journal Volume* **158** (2011), no 2, 225–246.
- 2010 **Length spectra and degeneration of flat metrics**,  
*with Moon Duchin and Christopher Leininger*,  
*Inventiones Mathematicae*, **182**, (2010) no. 2, 231–277.
- 2009 **Divergence rate of geodesics in Teichmüller space and mapping class groups**,  
*with Moon Duchin*,  
*Geometric and Functional Analysis* **19** (2009), no. 3, 722–742.
- 2009 **Covers and the curve complex**,  
*with Saul Schleimer*,  
*Geometry & Topology* **13** (2009), no. 4, 2141–2162.
- 2008 **Lines of minima and Teichmüller geodesics**,  
*with Young-Eun Choi and Caroline Series*,  
*Geometric and Functional Analysis* **18** (2008), no. 3, 698–754.

- 2008 **Lines of minima are uniformly quasigeodesic**,  
with Young-Eun Choi and Caroline Series,  
Pacific Journal of Mathematics **237** (2008), no. 1, 21–44.
- 2007 **Comparison between Teichmüller and Lipschitz metrics**,  
with Young-Eun Choi,  
Journal of the London Mathematical Society (2) **76** (2007), no. 3, 739–756.
- 2007 **A combinatorial model for the Teichmüller metric**,  
Geometric and Functional Analysis **17** (2007), no. 3, 936–959.
- 2007 **Thick-thin decomposition for quadratic differentials**,  
Mathematical Research Letters **14** (2007), no. 2, 333–341.
- 2005 **A characterization of short curves of a Teichmüller geodesic**,  
Geometry & Topology **9** (2005), 179–202.

## Invited Talks

### Recent Conferences

- July 2020 **Online**, *Australian Geometric Topology Webinar*.
- April 2020 **Online**, *Billiards and Surfaces à la Teichmüller and Riemann*.
- January 2020 **Tsinghua Sanya International Mathematics Forum**, *Sanya, China*, Frontier of Analysis and Geometry of Teichmüller Spaces.
- June 2019 **Bowdoin College**, *Brunswick*, Boundaries of Random Walks and Applications.
- August 2018 **Fields Institute**, *Toronto*, Workshop on Geometry of Teichmüller Space.
- July 2018 **Peking University**, *Beijing*, A conference on Geometric Group Theory.
- June 2018 **Fields Institute**, *Toronto*, 2018 Fields Institute Annual General Meeting.
- May 2018 **Stanford University**, *Palo Alto*, The Mathematical Legacy of Maryam Mirzakhani.
- April 2018 **University of Warwick**, *Coventry*, Geometry of Teichmüller space and mapping class groups.
- Nov. 2017 **ICTS**, *Bangalore*, Surface group representation and geometric structures.
- Sept. 2017 **Ventotene**, *Italy*, Moduli Space.
- August 2017 **Trinity College**, *Dublin*, Geometry and Dynamics of Moduli Spaces.
- June 2017 **Newton Institute**, *Cambridge*, Group actions and cohomology in non-positive curvature.
- May 2017 **Vanderbilt University**, *Vanderbilt*, Shank Conference.
- April 2017 **University of Arkansas**, *Fayetteville*, Redbud Topology Conference.
- Feb. 2017 **CIRM**, *Marseille*, Dynamics and Geometry in the Teichmüller Space.
- October 2016 **MSRI**, *Berkeley*, Geometry of mapping class groups and  $\text{Out}(\mathbb{F}_n)$ .
- July 2016 **KAIST**, *Daejeon, Korea*, Geometric Topology Fair.
- May 2016 **ICERM**, *Providence*, Effective and Algorithmic Methods in Hyperbolic Geometry and Free Groups.
- May 2016 **Casa Matemática Oaxaca**, *Oaxaca, Mexico*, Flat Surfaces and Dynamics of Moduli Space.

- Jan. 2016 **African Institute for Mathematical Sciences**, *Cape Town*, Quadratic Differentials.  
Sept. 2015 **Mathematical Institute**, *Oxford, UK*, Geometry and Dynamics on Moduli Spaces.  
June 2015 **Inter-University Centre Dubrovnik**, *Dubrovnik, Croatia*, Dubrovnik VIII - Geometric Topology, Geometric Group Theory & Dynamical Systems.  
May 2015 **University of Texas at Austin**, *Austin*, Workshop on Mapping Class Groups and  $\text{Out}(\mathbb{F}_n)$ .

#### Colloquia (Excluding job talks)

- 2019 **University of Toronto**.  
2017 **Pittsburgh University**.  
2017 **University of Michigan**.  
2014 **University of Oklahoma**.  
2014 **University of Indiana**.  
2011 **University of Arkansas**.  
2011 **Rice University**.

#### Seminars

Brown University, California Institute of Technology, Cornell University, CUNY Graduate Center, Columbia University, Harvard University, Institut des Hautes Études Scientifiques, Joint USC-UCLA-Caltech, Kyoto University, McGill University, Osaka University, Oklahoma State University, Princeton University, Rutgers University, South China University of Technology, Stanford University, Sun Yat-Sen University, SUNY at Stony Brook, Tufts University, Université de Rennes, Université Paris-Sud, University of Connecticut, University of Maryland, University of California at Davis, University of California at Santa Barbara, University of Chicago, University of Illinois at Chicago, University of Illinois at Urbana-Champaign, University of Michigan, University of Southern California, University of Seoul, University of Texas Austin, University of Toronto, University of Utah, University of Warwick, Yale University.

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## Supervision

#### Graduate students

- Current Ali Pirhosseinloo  
Current Assaf Bar-Natan  
Current Thomas Dykes  
Current Pooya Honaryar  
Current Ivan Telpukhovskiy  
Current Yvon Verberne  
2013 Funda Gültepe, University of Oklahoma  
2013 Yoe Alexander Herrera Jaramillo, University of Oklahoma

### Undergraduate

- 2020 Siyan He, (Supported by UT Work Study Program)
- 2019 Curtis Grant, (Supported by NSERC)
- 2018 Ci Zhang, (Supported by UT Work Study Program)
- 2018 Jordan Hofmann, (Supported by NSERC)
- 2017 Adriano Pacifico, (Supported by NSERC)

### Highschool Mentorship

- 2019 Anna Krokline
- 2018 Clare Radnoff
- 2016 Gemma Zhang
- 2015 Abtin Ameri
- 2015 Andrey Khesin
- 2014 Coco Zhang

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## Teaching Experience

### Undergraduate courses

Pre-calculus, Single and Multivariable Calculus at all levels, Honors Calculus, Ordinary and Partial Differential Equations, Physical Mathematics, Discrete Mathematics, Logic and Proofs, Linear Algebra, Abstract Algebra, Probability, Graph theory, Complex Analysis, General Topology, Knot Theory, Problem Solving Seminar, and Seminar in Mathematics.

### Graduate courses

Algebra for Teachers, Geometry for Teachers, Introduction to Hyperbolic Geometry, Algebraic Topology, Differential Topology, Combinatorial Methods and Geometric Literacy.

### Topics courses

Kleinian groups, Introduction to Teichmüller Space, Currents and Counting problems, Geometry and Dynamics in Teichmüller Space Equipped with the Thurston Metric

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## Recent Professional Activities

- May 2019 Co-organizer of the workshop *Flat Surfaces and Dynamics of Moduli Space,II*, Oaxaca.
- Fall 2018 Co-organizer of the Thematic Program on *Teichmüller Theory and its Connections to Geometry, Topology and Dynamics* Fields Institute, Toronto.
- Nov 2018 Co-organizer for the *Fields Symposium in honor of Maryam Mirzakhani*, Fields Institute, Toronto.
- 2018-present Associate Editor for *The Journal of Iranian Mathematical Society*
- 2016-2019 Associate Editor for *The Journal of the Ramanujan Mathematical Society*
- 2008-2018 Mentor for Association for Women in Mathematics (AWM) Mentor Network

## Collaborators

- Yael Algom-Kfir (University of Haifa)
- Mladen Bestvina (University of Utah)
- Mathew Clay (University of Arkansas)
- Young-Eun Choi
- Moon Duchin (Tufts University)
- Alex Eskin (University of Chicago)
- W Patrick Hooper (CUNY City College)
- Camillie Horbez (CNRS, Laboratoire de Mathématiques d'Orsay)
- Fanny Kassel (IHES)
- Misha Kapovich (Univ. of California Davis)
- Anna Lenzhen (Rennes)
- Howard Masur (University of Chicago)
- Maryam Mirzakhani (Stanford University)
- Yulan Qing (University of Toronto)
- Nikita Selinger (University of Alabama)
- Caroline Series (University of Warwick)
- Jing Tao (University of Oklahoma)
- Michael Yampolsky (University of Toronto)
- Juliette Bavard (Rennes)
- Jeffery Brock (Yale University)
- Camille Horbez (Orsay)
- Spencer Dowdall (Vanderbilt University)
- David Dumas (Univ. of Illinois at Chicago)
- Maxime Fortier Bourque (Univ. of Glasgow)
- Eriko Hironaka (Florida State University)
- Eriko Hironaka (Florida State University)
- Ilya Kapovich (CUNY Hunter College)
- Christopher J. Leininger (Rice University)
- Kathryn Mann (Cornell University)
- Babak Modami (SUNY Stony Brook)
- Mahan Mj (Tata Institute)
- Anja Randecker (University of Toronto)
- Saul Schleimer (University of Warwick)
- Juan Souto (Rennes)
- Yvon Verberne (University of Toronto)