1. If you have a large class with many chairs and many students, how can you determine if there are the same number of chairs as studnets?

2. Imagine a Grand Hotel with a infinite number rooms numbered $1, 2, 3, \ldots$. On this particular night, the hotel is completely full. Late in the evening, you arrive at the hotel and inquire about a room. Although there is no vacancy the hotel manager tells you that since this is an infinite hotel she can easily make room for you! You're delighted, but confused. If the infinite hotel is completely filled with an infinite number of guests, how does the manager go about securing a room for you?

3. Another night, another infinitely full hotel with no vacancy. This time the manager looks out the window and sees an infinitely long bus filled with an infinite number of passengers each in quest of a room. Our math savvy manager jumps up and begins preparing an infinite number of rooms for the infinite number of new guests. How does she do this?

4. As you can imagine this hotel is a huge success, so it's no wonder that one night our manager looks out the window to see an infinite number of busses all filled with an infinite number of room-seeking guests!! Can she make room for an infinite number of infinite people in the infinite hotel?

5. Which of these sets has more elements?
   - The set of natural numbers $\mathbb{N} = \{1, 2, 3, \ldots\}$.
   - The set of even numbers.
   - The set of prime numbers.
   - The set of positive rational numbers $\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, \quad p, q \in \mathbb{N}, \quad q \neq 0 \right\}$.
   - The set of binary sequences $\left\{ a_1 a_2 a_3 \ldots \,\middle|\, \quad a_i = 0 \text{ or } 1 \right\}$.
   - The set of subsets of $\mathbb{N}$.

6. Alice and Bob each have a box that can fit infinitely many balls. At each turn they put 10 balls in the box and remove one. The first batch of balls are labled $1 - 10$, the second batch $11 - 20$ etc.
   - At the n-th turn Alice removes the last ball of the last batch, namely, the ball labeled $10n$.
   - At the n-th turn Bob removes the ball with the lowest label, namely the ball labeled n.

   After infinitely many turns, how many balls do they have?

7. Achilles is in a footrace with the tortoise. Achilles allows the tortoise a head start of 100 meters, for example. After some finite time, Achilles will have run 100 meters, bringing him to the tortoise's starting point. During this time, the tortoise has run a much shorter distance, say 2 meters. It will then take Achilles some further time to run that distance, by which time the tortoise will have advanced farther; and then more time still to reach this third point, while the tortoise moves ahead. Thus, whenever Achilles arrives somewhere the tortoise has been, he still has some distance to go before he can even reach the tortoise. Is this a correct argument?

8. Calculate the following infinite sums:
   - $1 + \dfrac{1}{2} + \dfrac{1}{4} + \cdots + \dfrac{1}{2^n} + \ldots.$
   - $\dfrac{2}{3} + \dfrac{4}{9} + \cdots + \dfrac{2^n}{3^n} + \ldots.$
   - $1 + \dfrac{1}{2} + \dfrac{1}{3} + \cdots + \dfrac{1}{n} + \ldots.$