

MAT344 H1 S:INTRODUCTION TO COMBINATORICS

Winter 2023

Instructor:	Kasra Rafi	Time:	Tu 12-2 Th 12-1
Email:	rafi@math.toronto.edu	Place:	MP 202
Office hrs:	Tu and Th 11-12	Office:	BA 6236

Quercus Course Page: <https://www.math.toronto.edu/~rafi/MAT344.html>

What is Combinatorics?

Combinatorics is a field of mathematics concerned with counting and finite structures. Combinatorics is a very diverse subject that has many applications to other fields of mathematics and computer science. The goal of this course is to introduce you to a variety of techniques and ideas that will help you solve a wide range of problems. For example, you may know the algebraic identity $1 + 2 + \dots + n = n(n+1)/2$. A combinatorial proof of this identity can be given by simply counting a set of objects in two different ways, and proofs of this sort are very transparent and enlightening. The topics we will cover include graphs, recurrences, induction, generating functions, inclusion-exclusion and probability.

Textbook

We will use Applied Combinatorics by M.T. Keller and W.T. Trotter, 2017 edition as our main textbook. Available for free at

<http://www.rellek.net/appcomb/>

Quizzes and Homework

Problem Sets will be assigned Weekly. In mathematics, one learns through practice, i.e. by doing lots of problems. There are several other books noted at the end of this syllabus where you can find more problems, if the ones in the main text are not enough. There will be a weekly quiz in the tutorial which will consist of one of the assigned problems.

Assessment

Quizzes 25% The three lowest quizzes will be dropped

Midterms 30% Two 1hr midterms Feb 16 and Mar 9. Midterm grade will be max of the two.

Final Exam 45% To be Scheduled

Regrades

If you would like to request a regrade on an assessment, you will need to make a written submission explaining what you believe was marked incorrectly. TAs will not discuss grading in tutorials. If an assessment is regraded, it will be carefully scrutinized, and your mark may go down.

Calculator policy

Calculators are not necessary (or helpful) for this kind of material; for that reason, they are not allowed during quizzes or exams. We would much prefer that you write your solutions in the form of 17^6 rather than 24137569 (for example).

Illnesses

If you have a serious long term illness that prevents you from attending multiple quizzes, email the instructor. Missing both the midterms for medical reasons requires a verification of illness form, and the weight will be put on the final exam. The form can be found at:

<http://www.illnessverification.utoronto.ca>

and must be filled out by a doctor. Submitting a forged medical note is an academic offence.

Tutorials

Tutorials start the second week of classes. They give you the opportunity to work in-depth on problems in small groups with TA guidance. The problems will require you to apply course concepts and justify your solutions to others. You must attend the tutorials because there will be a quiz each week which counts 25% towards your final grade. Tutorials will be your best opportunity to practice solving novel questions under time constraints, like you would on a test, and get immediate feedback on your solutions from peers and TAs. Solutions will be posted on Quercus after all tutorials have finished.

Role in your program

Prerequisites: MAT 223. We will expect you to have a solid understanding of algebraic manipulations, solving linear systems and set notation, as well as some familiarity with writing proofs. Programs recommending MAT344: Applied Math Specialist, Focus in Theory of Computation, Math Applications in Economics and Finance, Math & Applications Teaching Specialist, Math Major. Role in your program: We hope to explain the connection between enumeration and algorithm complexity, and motivate pedagogical questions which can be solved by combinatorial methods, while maintaining the standards of 300-level mathematics courses. These standards include clear communication in written proofs.

Other books: You may want to learn about other combinatorial topics that we don't have time to discuss, like Latin squares or block designs, or more about specific topics. Some other books you may find relevant are:

- Combinatorics Through Guided Discovery by K. P. Bogart is an inquiry-based learning book that covers many of the topics of the course, and is a great source for insightful problems. Available for free at
<http://bogart.openmathbooks.org>
- Combinatorics by J. Morris is a more traditional theorem-proof style textbook that includes topics like Latin squares and designs. Available for free at
<http://www.cs.uleth.ca/~morris/Combinatorics/Combinatorics.html>
- Combinatorics and Graph Theory by D. Guichard is another more traditional style textbook with an emphasis on Graph theory. Available for free at
https://www.whitman.edu/mathematics/cgt_online/book/
- generatingfunctionology by H. S. Wilf is a book focused on generating functions and their applications. Available for free at
<https://www.math.upenn.edu/~wilf/DownldGF.html>

Policy on Missed Term Work As flexibility for missed quizzes have been built into the marking scheme, missed quizzes will not be accepted.

Please note that Verification of Illness forms (i.e. doctor's note) are temporarily not required. Students who are absent from class for any reason (e.g. COVID, flu or any other illness, injury or family situation) and who require consideration for missed academic work should report their absence through the online absence declaration. This is available on ACORN under the Profile and Settings menu.

Sage: You can find Sage Math cells at <https://sagecell.sagemath.org/> for quick computations. Sage is based on Python, and many helpful articles are available. The online version of the textbook also has embedded Sage Math cells with some code already written to demonstrate the computations (for example, in http://www.rellek.net/book/s_intro_number.html).

Calendar

Strings, Sets, and Binomial Coefficients. Sections 2.1-2.6 Week 1 (Jan 9–13)
Introduction to enumeration, permutations, combinatorial proofs and the binomial theorem.

Well-ordering, Recurrence and Induction. Sections 3.1-3.8 Week 2 (Jan 16–20)
Our first look at recurrence relations, motivating the formal proof system of induction.

Strong Induction, Pigeonhole Principle and Complexity. Section 3.9, and Chapter 4
 Week 3 (Jan 23–27)
A variant of induction suitable for recurrences; a famous existence theorem; and an introduction to growth rates of functions, motivated by how difficult certain enumeration problems are.

Graph basics. Sections 5.1-5.4 Week 4 (Jan 30–Feb 3)
Notation and basic properties of graphs.

Counting graphs. Sections 5.5-5.7 Week 5 (Feb 6–10)
Both exact and asymptotic enumerations for certain graphs of given size.

Inclusion-Exclusion. Chapter 7 Week 6 (Feb 13–17)
A counting principle that applies to collections of intersecting sets. Applications of the Inclusion-exclusion formula.

Midterm 1. Up to and including Section 5.7 (Feb 16)

Generating functions. Chapter 8 Week 7 (Feb 27–Mar 3)
A bookkeeping method to store information about sequences in a useful way.

Recurrences. Chapter 9 Week 8 (Mar 6–10)
Methods to solve basic recurrence equations.

Midterm 2. Up to and including Chapter 8 (Mar 9)

Probability. Sections 10.1-10.3 Week 9 (Mar 13–17)
Basic concepts and their relation to enumeration.

Discrete random variables. Sections 10.4-10.6 Week 10 (Mar 20–24)
Using probability to compute expectation.

Graph algorithms. Chapter 12 Week 11 (Mar 27–31)
Finding minimal spanning trees in graphs and directed graphs.

Network flows. Chapters 13 Week 12 (Apr 3–6)
Networks and the Max-flow min-cut theorem.

Institutional Policies and Support

Academic Integrity

All suspected cases of academic dishonesty will be investigated following procedures outlined in the Code of Behaviour on Academic Matters

<https://governingcouncil.utoronto.ca/secretariat/policies/code-behaviour-academic-matters-july-1-2019>

If you have questions or concerns about what constitutes appropriate academic behaviour or appropriate research and citation methods, please reach out to your Course Instructor. Note that you are expected to seek out additional information on academic integrity from me or from other institutional resources for example, the University of Toronto website on Academic Integrity

<http://academicintegrity.utoronto.ca/>

Accessibility

The University provides academic accommodations for students with disabilities in accordance with the terms of the Ontario Human Rights Code. This occurs through a collaborative process that acknowledges a collective obligation to develop an accessible learning environment that both meets the needs of students and preserves the essential academic requirements of the University's courses and programs. Students with diverse learning styles and needs are welcome in this course. If you have a disability that may require accommodations, please feel free to approach your Course Instructor and/or the Accessibility Services office as soon as possible. The sooner you let us know your needs the quicker we can assist you in achieving your learning goals in this course.

Link to Accessibility Services website

<https://studentlife.utoronto.ca/department/accessibility-services/>

Equity, Diversity and Inclusion

The University of Toronto is committed to equity, human rights and respect for diversity. All members of the learning environment in this course should strive to create an atmosphere of mutual respect where all members of our community can express themselves, engage with each other, and respect one another's differences. U of T does not condone discrimination or harassment against any persons or communities.

Important Academic Dates & Deadlines

The academic dates include enrolment dates, drop deadlines, exam periods, petition deadlines and more.

<https://www.artsci.utoronto.ca/current/dates-deadlines/academic-dates>

Other Academic and Personal Supports

- Writing Centre: <https://writing.utoronto.ca/writing-centres/arts-and-science/>
- U of T Libraries: <https://onesearch.library.utoronto.ca/>
- Feeling Distressed? <https://studentlife.utoronto.ca/task/support-when-you-feel-distressed/>
- Academic Success Centre: <https://studentlife.utoronto.ca/department/academic-success/>
- College/Faculty Registrars: <https://future.utoronto.ca/current-students/registrars/>