Problem Set I, MAT 382, Fall 2020

Due October 19, 2020

Read Chapters 4 and 5 from the text book.

1. Chapter 4: Exercises 4, 8, 9.
2. Chapter 5: Exercises 2, 6, 11, 19.
3. Project 2 in Chapter 5.
4. Correct the Step 1 in the proof of Nielsen-Schreier theorem. That is, prove that if a group G acts freely on a tree T and v is a vertex in T, then there is a tiling of T with connected tiles T_{gv} containing the vertex $g \cdot v$ such that

   (a) $T_{gv} = g \cdot T_v$.

   (b) The union $\cup_{g \in G} T_{gv}$ covers T.

   (c) Different tiles intersect in at most one point.

   You may assume that every vertex in T has finite valence.

5. Consider the following subgroup of $F_2 = \langle a, b \rangle$:

$$H = \left\langle a^3 b, \overline{a}bab, a^2\overline{b}a \right\rangle$$

   (a) Show that $H \neq F_2$?

   (b) What is the rank H?

   (c) Is $w = (ab)^3$ an element of H?