I) In the plane, n lines are given ($n \geq 3$), no two of them parallel. Through every intersection of two lines there passes at least an additional line. Prove that all lines pass through one point.

II) Numbers are placed on a chessboard. It is given that any number is equal to an arithmetical mean of its neighbours by a side. Prove that all numbers are equal.

III) Let us consider a point P inside a convex polygon and drop the perpendiculars from P to each side or its extension. Prove that at least one of the perpendiculars meets a corresponding side.

IV) Let A be a set of $2n$ points in the plane, no three of which are collinear. Suppose that n of them are red and the remaining n blue. Prove or disprove: There are n closed straight line segments, no two with a point in common, such that the endpoints of each segment are points of A. having different colors.

V) Let $f(x)$ be a polynomial of degree n with real coefficients and such that $f(x) > 0$ for every real number x. Show that $f(x) + f'(x) + \cdots + f^{(n)}(x) > 0$ for all real x. (Recall that $f^{(i)}$ denotes the i–th derivative of f).

VI) Show that there exists a rational number, $\frac{c}{d}$, with $d < 100$, such that

$$\left\lfloor k\frac{c}{d} \right\rfloor = \left\lfloor k\frac{73}{100} \right\rfloor \qquad \text{for } k = 1, \ldots, 100.$$

VII) Let S be a nonempty set of integers such that

(i) the difference $x - y$ is in S whenever x and y are in S, and

(ii) all multiples of x are in S whenever x is in S.

a) Prove that there is an integer d in S such that S consists of all multiples of d.

b) Show that part a) applies to the set $\{ma+nb \parallel m, n \in \mathbb{Z}\}$ and that the resulting d is $\gcd(a, b)$.

c) Prove that any two successive Fibonacci numbers F_n, F_{n+1}, $n > 2$ are relatively prime.

VIII) There are n points given in the plane. Any three of the points form a triangle of area ≤ 1. Show that all n points lie in a triangle of area ≤ 4.

IX) In every tetrahedron, there are three edges meeting at the same vertex from which a triangle can be constructed.

X) There are n identical cars on a circular track. Together they have just enough gas for one car to complete a lap. Show that there is a car which can complete a lap by collecting gas from the other cars on its way around.

XI) Can you choose 1983 pairwise distinct positive integers less than 100000, such that no three are in arithmetic progression?