I) Fifteen chairs are placed around a circular table and there is a name card placed in front of each chair. The guests fail to see these cards until after they sat down and it turns out no one is sitting in front of her own card. Prove that the table can be rotated so that at least two of the guests are simultaneously correctly seated.

II) A rectangle is said to be *part whole* if the length of at least one of its sides is a whole number. Prove that if a rectangle R can be partitioned into part whole subrectangles, then R is part whole.

III) (a) Can you cover an $8 \times 8$ chessboard with 21 rectangles of size $3 \times 1$ and a single extra $1 \times 1$ square?

(b) Can you pack 125 boxes of size $4 \times 2 \times 1$ inside one $10 \times 10 \times 10$ cube?

IV) Can you draw a quadrilateral (non-convex!) and an additional straight line, such that the straight line cuts through the interior of each of the quadrilateral's edges? How about a pentagon?

V) Let $n$ be an odd integer and let $A$ be a symmetric $n \times n$ *Latin matrix* - every row and every column in $A$ is a permutation of $\{1, 2, \ldots, n\}$. Show that the diagonal of $A$ is also a permutation of $\{1, 2, \ldots, n\}$.

VI) (a) 10 prisoners are made to stand in a line by an evil warden, who also places either a black hat or a white hat on the head of each prisoner. Each prisoner can see all the hats in front of her, but not her own hat or the hats behind her. The evil warden than asks each prisoner for the colour of her hat, starting from the last one who sees all but herself and ending with the first one who sees no one. The prisoners are allowed to make at most one mistake; if they make more than one mistake, they are all executed. Assuming the prisoners know in advance that they will be subject to this game, can they devise a strategy that will allow them to survive?

(b) Our evil warden makes infinitely many prisoners stand in a circle so that each one can see the colour of the hats (black or white) on all heads but herself. All at once, each has to shout the colour of the hat on her head; if only finitely many get it wrong, they are all freed. But if more than just finitely many get it wrong, well, you know what happens in prisoner problems that involve an evil warden. Assuming they had the day before to devise a strategy, can they survive?

VII) Let $n$ be odd and $\{\sigma_1, \ldots, \sigma_n\}$ be a permutation of $\{1, 2, \ldots, n\}$. Prove that the following product is even:

$$(\sigma_1 - 1)(\sigma_2 - 2) \ldots (\sigma_n - n)$$

VIII) Show that for every positive integer $a$, the equation $x^2 - y^2 = a^3$ has solutions with $x, y \in \mathbb{Z}$.

IX) $n$ playing cards are placed in a sequence in front of you, some face up and some face down. At any step, you can flip a face up card and any number of cards on its right, no matter which face they are showing. Show that after finitely many steps you will necessarily be facing a sequence of face-down cards.

X) Prove that if $n > 1$, the sum $1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not an integer.