

- I) Prove that the graph of a cubic equation $y = ax^3 + bx^2 + cx + d$ is symmetric about its point of inflection,
- II) A right-angled triangle with integer sides has area divisible by three.
- III) A function on the plane has the property that the sum of its values on the corners of any square (of any size and orientation) is zero. Prove that the function is the zero function.
- IV) Consider the following two player game. Each player takes turns placing a penny on the surface of a rectangular table. No penny can touch a penny which is already on the table. The table starts out with no pennies. The last player who makes a legal move wins. Does any player have a winning strategy?
- V) 15 pennies are laid on a table forming a triangle. The faces of each of them are colored either black or white. Prove that there exist three pennies of the same color whose centers are the vertices of an equilateral triangle.
- VI) Four bugs are situated at each vertex of a unit square. Suddenly, each bug begins to chase its counterclockwise neighbor. If the bugs travel at 1 unit per minute, how long will it take for the four bugs to crash into one another?
- VII) Prove
- $$(a + b)(b + c)(c + a) \geq 8abc$$
- VIII) Given a point (a, b) with $0 < b < a$, determine the minimum perimeter of a triangle with one vertex at (a, b) , one on the x -axis, and one on the line $y = x$. You may assume that a triangle of with one vertex at minimum perimeter exists.
- IX) A billiard ball strikes the ray BC at point C , with angle of incidence α as shown. The billiard ball continues its path, bouncing off rays BA and BC according to the rule "angle of incidence equals angle of reflection." If $AB = BC$, determine the number of times the ball will bounce off the two line segments (including the first bounce, at C). Your answer will be a function of α and β .

