I) One morning it started snowing at a heavy and constant rate. A snowplow started out at 8:00 A.M. At 9:00 A.M. it had gone 2 miles. By 10:00 A.M. it had gone 3 miles. Assuming that the snowplow removes a constant volume of snow per hour, determine the time at which it started snowing.

II) Wires are strung from the top of each of two poles to the base of the other. What is the height from the ground where the two wires cross?

III) Use algebra to support your answers to each of the following.

(a) If the price of stocks you are holding goes up by %5 during the first week and then goes down during the second week have you made money or lost money? What if the price goes down by %5 during the first week and then goes up during the second week?

(b) A car travels from A to B at the rate of 40 miles per hour and then returns from B to A at the rate of 60 miles per hour. Is the average rate for the round trip more or less than 50 miles per hour?

(c) You are given a cup of coffee and a cup of cream, each containing the same amount of liquid. A spoonful of cream is taken from the cup and put into the coffee cup, then a spoonful of the mixture is put back into the cream cup. Is there now more or less cream in the coffee cup than coffee in the cream cup?

(d) Imagine that the earth is a smooth sphere and that a string is wrapped around it at the equator Now suppose that the string is lengthened by six feet and the new length is evenly pushed out to form a larger circle just over the equator. Is the distance between the string and the surface of the earth more or less than one inch?

IV) Prove that the sequence

$$11, \quad 111, \quad ,1111, \quad 1111, \ldots$$

contains no squares.

V) An (ordered) triple (x_1, x_2, x_3) of positive irrational numbers with $x_1 + x_2 + x_3 = 1$ is called balanced if each $x_i < \frac{1}{2}$. If a triple is not balanced, say if $x_j > \frac{1}{2}$, one performs the following "balancing act":

$$B(x_1, x_2, x_3) = (x_1', x_2', x_3'),$$

where $x_i' = 2x_i$ if $i \neq j$ and $x_j = 2x_j - 1$. If the new triple is not balanced, one performs the balancing act on it. Does continuation of this process always lead to a balanced triple after a finite number of performances of the balancing act?

VI) A straight line cuts the asymptotes of a hyperbola in points A and B and the curve in points P and Q. Prove that $AP = BQ$.

VII) Arrange the numbers $1, 2, \ldots, n$ consecutively (say, clockwise) about the circumference of a circle. Now, remove number 2 and proceed clockwise by removing every other number, among those that remain, until only one number is left. (Thus, for n = 5, numbers are removed in the order 2, 4, 1, 5, and 3 remains alone.) Let $f(n)$ denote the final number which remains. Describe the function f. Hint: First show that

$$f(2n) = 2f(n) - 1 \quad \text{and} \quad f(2n+1) = 2f(n) + 1.$$