

- I) The Euclidean plane is divided into regions by drawing a finite number of straight lines. Show that it is possible to color each of these regions either red or blue in such a way that no two adjacent regions have the same color.
- II) Two poles, with heights a and b , are a distance d apart (along level ground). A guy wire stretches from the top of each of them to some point P on the ground between them. Where should P be located to minimize the total length of the wire?
- III) A rectangular room measures 30 feet in length and 12 feet in height, and the ends are 12 feet in width. A fly, with a broken wing, rests at a point one foot down from the ceiling at the middle of one end. A smudge of food is located one foot up from the floor at the middle of the other end. The fly has just enough energy to walk 40 feet. Show that there is a path along which the fly can walk that will enable it to get to the food.
- IV) Let a and b be given positive real numbers with $a < b$. If two points are selected at random from a straight line segment of length b , what is the probability that the distance between them is at least a ?
- V) Seventeen people correspond by mail with one another each one with all the rest. In their letters only three topics are discussed. Each pair of correspondents deals with only one of the topics. Prove that there are at least three people who write to each other about the same topic.
- VI) Prove the following inequality:
- $$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n-1} \geq \log 2$$
- VII) A classroom has 5 rows of 5 desks per row. The teacher requests each pupil to change his seat by going either to the seat in front of him, the one behind him, the one to his left, or the one to his right (of course, not all these options are open to all the pupils). Determine whether this directive can be carried out.
- VIII) A cyclist is traveling along path. On day 1, he leaves point A at 10 am and arrives at point B at 4 pm. On day 2, he leaves point B at 10 am and arrives at point A at 4 pm. Show that there is a time of the day when, on both days, he was at the same place at that time.