

I) Suppose some blue socks and the same number of red socks are in a drawer. Suppose it turns out that the minimum number of socks I must pick in order to be sure of getting at least one pair of the same color is the same as the minimum number I must pick in order to be sure of getting at least two socks of different colors. How many socks are in the drawer?

II) In the town of Podunk, the following facts are true:

- (i) No two inhabitants have exactly the same number of hairs.
- (ii) No inhabitant has exactly 518 hairs.
- (iii) There are more inhabitants than there are hairs on the head of anyone inhabitant.

What is the largest possible number of inhabitants of Podunk?

III) A man owned no watch, but he had an accurate clock which, however, he sometimes forgot to wind. Once when this happened he went to the house of a friend, passed the evening with him, went back home, and set his clock. How could he do this without knowing beforehand the length of the trip?

IV) There is an island in which certain inhabitants called "knights" always tell the truth, and others called "knaves" always lie. It is assumed that every inhabitant of the island is either a knight or a knave.

Three of the inhabitants - A, B, and C - were standing together in a garden. A stranger passed by and asked A, "How many knights are among you?" A answered, but rather indistinctly, the stranger could not hear. The stranger asks B, "What did A say?" B replies, "A said that there is one knight among us." Then C says, "Don't believe B; he is lying!" What are B and C?

V) Mr. McGregor, a London shopkeeper, phoned Scotland Yard that his shop had been robbed. Three suspects A,B,C were rounded up for questioning. The following facts were established:

- (i) Each of the men A,B,C had been in the shop on the day of the robbery, and no one else had been in the shop that day.
- (ii) If A was guilty, then he had exactly one accomplice.
- (iii) If B is innocent, so is C.
- (iv) If exactly two are guilty, then A is one of them.
- (v) If C is innocent, so is B.

Whom did Inspector Craig indict?

VI) Suppose you are an inhabitant of the island of knights and knaves. You fall in love with a girl there and wish to marry her. However, this girl has strange tastes; for some odd reason she does not wish to marry a knight; she wants to marry only a knave. But she wants a rich knave, not a poor one. (We assume for convenience that everyone there is classified as either rich or poor.) Suppose, in fact, that you are a rich knave. You are allowed to make only one statement to her. How, in only one statement, can you convince her that you are a rich knave?

VII) On the island of knights and knaves, three inhabitants A,B,C are being interviewed. A and B make the following statements:

A: B is a knight.

B: If A is a knight, so is C.

Can it be determined what any of A,B,C are?

VIII) The King of a small country invites 1000 senators to his annual party. As a tradition, each senator brings the King a bottle of wine. Soon after, the Queen discovers that one of the senators is trying to assassinate the King by giving him a bottle of poisoned wine. Unfortunately, they do not know which senator, nor which bottle of wine is poisoned, and the poison is completely indiscernible. However, the King has 10 prisoners he plans to execute. He decides to use them as taste testers to determine which bottle of wine contains the poison. The poison when taken has no effect on the prisoner until exactly 24 hours later when the infected prisoner suddenly dies. The King needs to determine which bottle of wine is poisoned by tomorrow so that the festivities can continue as planned. Hence he only has time for one round of testing. How can the King administer the wine to the prisoners to ensure that 24 hours from now he is guaranteed to have found the poisoned wine bottle?

The problems discussed in class

I) There is an island upon which a tribe resides. The tribe consists of 1000 people, with various eye colours. Yet, their religion forbids them to know their own eye color, or even to discuss the topic; thus, each resident can (and does) see the eye colors of all other residents, but has no way of discovering his or her own (there are no reflective surfaces). If a tribesperson does discover his or her own eye color, then their religion compels them to commit ritual suicide at noon the following day in the village square for all to witness. All the tribespeople are highly logical and devout, and they all know that each other is also highly logical and devout (and they all know that they all know that each other is highly logical and devout, and so forth).

II) A judge tells a condemned prisoner that he will be hanged at noon on one weekday in the following week but that the execution will be a surprise to the prisoner. He will not know the day of the hanging until the executioner knocks on his cell door at noon that day.

Having reflected on his sentence, the prisoner draws the conclusion that he will escape from the hanging. His reasoning is in several parts. He begins by concluding that the "surprise hanging" can't be on Friday, as if he hasn't been hanged by Thursday, there is only one day left - and so it won't be a surprise if he's hanged on Friday. Since the judge's sentence stipulated that the hanging would be a surprise to him, he concludes it cannot occur on Friday.

He then reasons that the surprise hanging cannot be on Thursday either, because Friday has already been eliminated and if he hasn't been hanged by Wednesday noon, the hanging must occur on Thursday, making a Thursday hanging not a surprise either. By similar reasoning he concludes that the hanging can also not occur on Wednesday, Tuesday or Monday. Joyfully he retires to his cell confident that the hanging will not occur at all.

The next week, the executioner knocks on the prisoner's door at noon on Wednesday – which, despite all the above, was an utter surprise to him. Everything the judge said came true.

Logical Paradox

- I) A crocodile, who has stolen a child, promises the father/mother that their child will be returned if and only if they correctly predict what the crocodile will do next, return the child or eat it. What should the parents do?
- II) This story concerns a caravan going through the Sahara desert. One night they pitched tents. Our three principle characters are A, B, and C. A hated C and decided to murder him by putting poison in the water of his canteen (this would be C's only water supply). Quite independently of this, B also decided to murder C, so (without realizing that C's water was already poisoned) he drilled a tiny hole in C's canteen so that the water would slowly leak out. As a result, several days later C died of thirst. The question is, who was the murderer, A or B? According to one argument, B was the murderer, since C never did drink the poison put in by A, hence he would have died even if A hadn't poisoned the water. According to the opposite argument, A was the real murderer, since B's actions had absolutely no effect on the outcome; once A poisoned the water, C was doomed, hence A would have died even if B had not drilled the hole.
- III) On April 1st Emile went to Raymond's bedroom and said: "Well, Raymond, today is April Fool's Day, and I will fool you as you have never been fooled before!" Raymond waited all day long for Emile to fool him, but he didn't. Late that night, his mother asked him, "Why don't you go to sleep?" He replied, "I'm waiting for Emile to fool me." His mother turned to Emile and said, "Emile, will you please fool the child!" Emile then turned to Raymond, and the following dialogue ensued:
- Emile: So, you expected me to fool you, didn't you?
Raymond: Yes.
Emile: But I didn't, did I?
Raymond: No.
Emile: But you expected me to, didn't you?
Raymond: Yes.
Emile: So I fooled you, didn't I?!
- IV) If you remove a single grain of sand from a heap, you still have a heap. Keep removing single grains, and the heap will disappear. Can a single grain of sand make the difference between heap and non-heap?
- V) Consider the following' paradox:

This sentence can never be proved.

The paradox is this: If the sentence is false, then it is false that it can never be proved, hence it can be proved, which means it must be true. So, if it is false, we have a contradiction, therefore it must be true. Now, we have just proved that the sentence is true. Since the sentence is true, then what it says is really the case, which means that it can never be proved. So how come I have just proved it?