I) Show that there must be at least two people in London with the same number of hairs on their heads.

II) Ten points are placed within a unit equilateral triangle. Show that there exists two points with distance at most $\frac{1}{3}$ apart.

III) What is the minimum number of cards that must be drawn from a standard deck to guarantee at least three cards all of the same suit?

IV) Prove that having 100 whole numbers, one can choose 15 of them so that the difference of any two is divisible by 7.

V) Every point in a plane is either red, green, or blue. Prove that there exists a rectangle in the plane such that all of its vertices are the same color.

VI) If you pick 17 integers from 8 to 39, then two of them must add up to 47.

VII) A 10 by 10 table is filled in with positive integers so that adjacent integers (i.e., integers are adjacent if their squares share a side) differ by 5 or less. Show that the table must contain two identical integers.

VIII) If you draw five points on the surface of an orange in permanent marker, then there is a way to cut the orange in half so that four of the points will lie on the same hemisphere (suppose a point exactly on the cut belongs to both hemispheres).

IX) Gary is training for a triathlon. Over a 30 day period, he pledges to train at least once per day, and 45 times in all. Then there will be a period of consecutive days where he trains exactly 14 times.

X) Suppose that we are given a sequence of $nm + 1$ distinct real numbers. Prove that there is an increasing subsequence of length $n + 1$ or a decreasing subsequence of length $m + 1$.